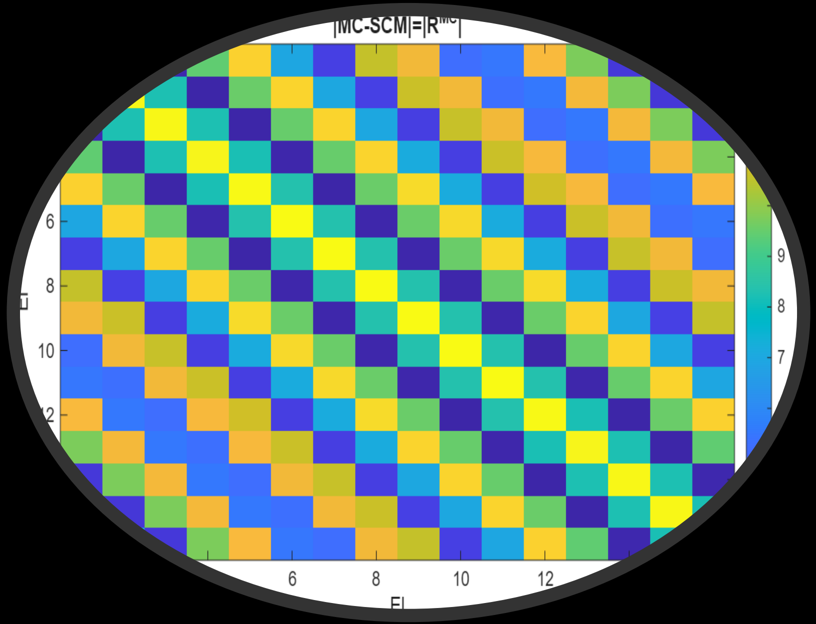
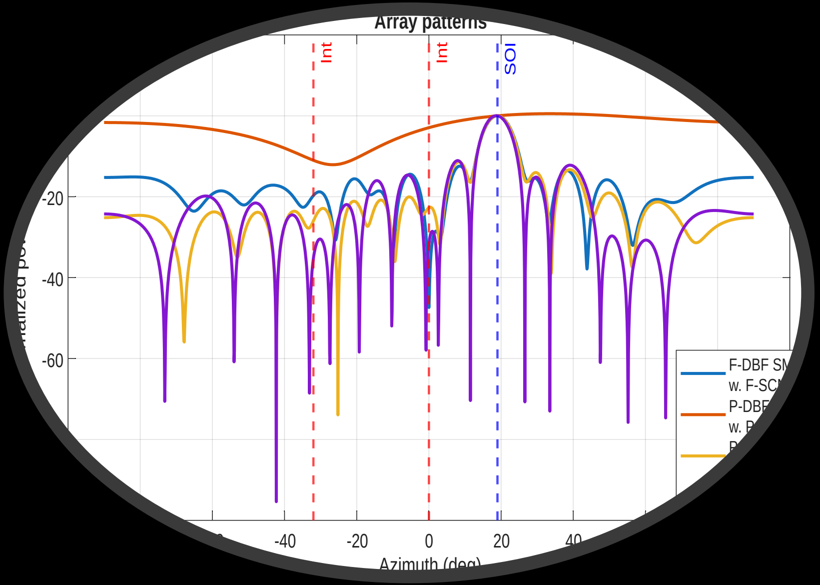
Hybrid SMI on ADI ADALM-Phasers: Structure-Aware Covariance Completion in MATLAB





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Our Work In One Slide

Develop theory and methods for *practical MVDR-SMI* on hybrid beamformers (HBF) that achieves *near-digital beamformer (DBF) performance*

Demonstrate our approach in practice by pushing the Phaser to achieve SINR performance approaching that of an 8-channel DBF

Motivation — Why Adaptive Beamforming (and Why Hybrid)

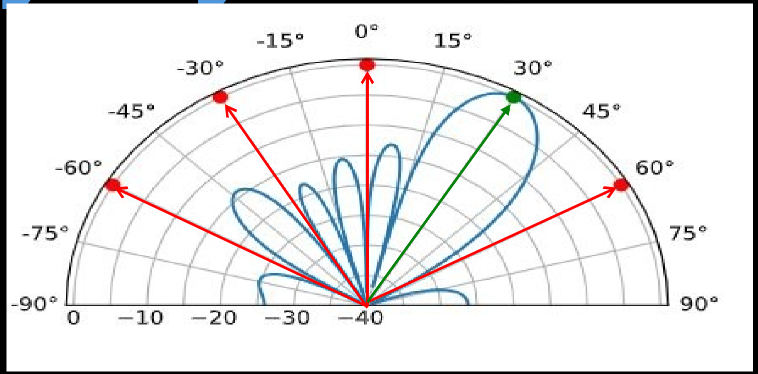


Fig 1: Adaptive Array's Pattern for unknown interferers (**red** dots) and SoI (**green** dot)

- Adaptive Beamforming (ADBF) Goal:
  Calculate array weights that capture the signal of interest (SoI) while suppressing unknown interference and noise

Minimum Variance Distortionless Response:
Minimize output power of a N element antenna array subject to unity gain at SoI using knowledge of the covariance matrix ($\boldsymbol{R} \in \mathbb{C}^{N\times N}$)

$$\min_{\mathbf{w}} \mathbf{w}^H\mathbf{R}\mathbf{w} \quad \text{subject to} \quad \mathbf{s}_{\theta_o}^H\mathbf{w} = \mathbf{1}$$

$$\mathbf{w_{mvdr}} = \frac{\mathbf{R}^{-1}\mathbf{s}_{\theta_o}}{\mathbf{s}_{\theta_o}^H\mathbf{R}^{-1}\mathbf{s}_{\theta_o}}$$

Sample Matrix Inversion (SMI):
Practical MVDR, uses Sample Covariance Matrix (SCM) ($\widehat{\boldsymbol{R}} \in \mathbb{C}^{N\times N}$) from data matrix **X**; K samples across N elements

$$\mathbf{w_{smi}} = \frac{(\hat{\mathbf{R}} + \delta\mathbf{I})^{-1}\mathbf{s}_{\theta_o}}{\mathbf{s}_{\theta_o}^H(\hat{\mathbf{R}} + \delta\mathbf{I})^{-1}\mathbf{s}_{\theta_o}}$$

$$\hat{\mathbf{R}} = \frac{1}{K}\mathbf{X}\mathbf{X}^{\mathbf{H}}$$

$$\mathbf{X} \in \mathbf{C}^{N\times K}$$

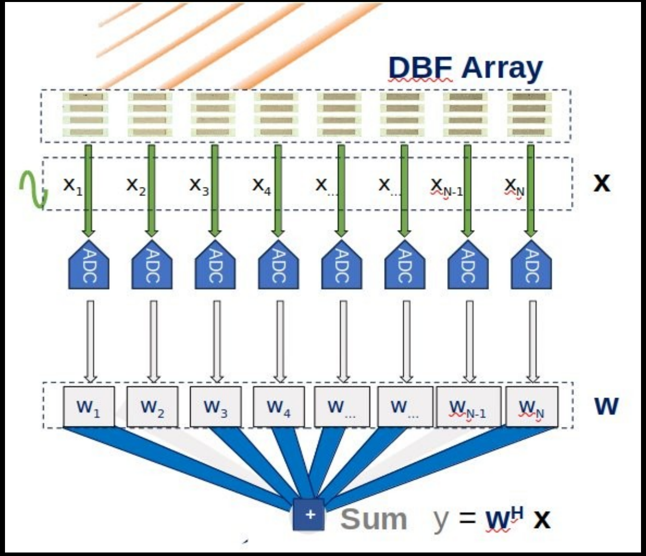
[N, K] = # of elements, # of snapshots/samples, s – steering vector, θ_0– direction of SOI, δ– regularization parameter

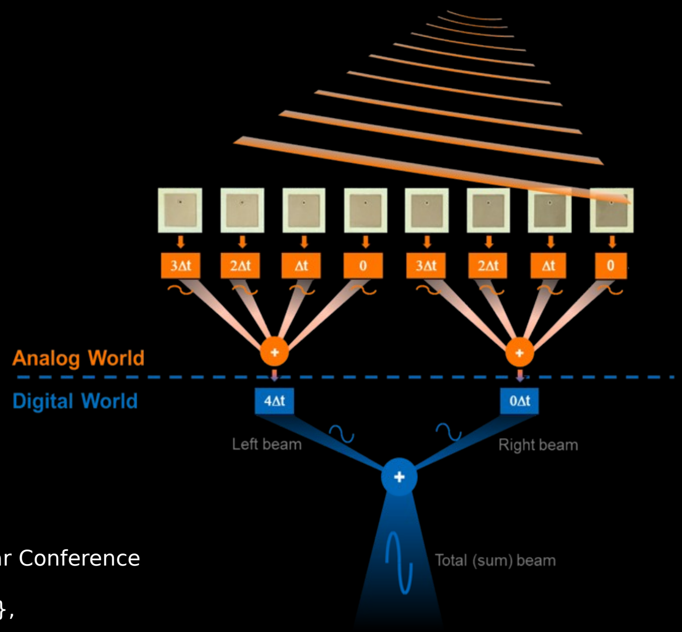
MVDR/SMI In Hybrid Beamformers (HBFs)

- DBF**:** N ADCs; Proven MVDR/SMI performance, High SWaP-C
- HBF**:** P ADCs (P<N); **L**ower SWaP-C

- Partial-DBFs (P-DBF)**:** DBF w. P ADCs,
- Full-DBF (F-DBF)**:** DBF w. N ADCs
- Variants of HBF exist (full-HBF, Partial-HBF and sub-array based HBF)

- The focus of this work is on sub-array based Hybrid Arrays (Phaser)
- Recent work adapts MVDR to HBF & shows near-DBF performance*





* B. K. Chalise and M. G. Amin, "Hybrid MVDR Beamformer for Subarray Architecture with Sparse Recovery and Manifold Optimization Methods," 2024 IEEE Radar Conference (RadarConf24), Denver, CO, USA, 2024, pp. 1-6, doi: 10.1109/RadarConf2458775.2024.10548853. keywords: {Manifolds;Array signal processing;Phase shifters;Interference;Radar;Hybrid power systems;Transceivers;Hybrid MVDR beamformer;subarray;manifold optimization;sparse signal recovery;massive MIMO},

Prior Work

- *Proposed a hybrid beamforming strategy to jointly compute N analog and P digital weights that approximate the MVDR solution of a N element DBF, assumes access to the covariance matrix ($\boldsymbol{R} \in \mathbb{C}^{N\times N}$)
- Weights constrained to unit-magnitude; phase is variable
- Analog weights solved on a Riemannian manifold using Manopt
- Reported results: H-MVDR (red line) performs ***slightly worse*** than F-DBFs, but better than P-DBFs

(1) $$\min_{\mathbf{w},\mathbf{W}_d\in\mathcal{F}} ||\mathbf{w}_o - \mathbf{W}_d^H \mathbf{w}||^2$$

(2) $$\mathbf{w} = \frac{1}{M}\mathbf{W}_d\mathbf{w}_o$$

(3) $$\max_{|[\mathbf{w}_d]_k|=1,\forall k} \mathbf{w}_d^H \mathbf{W}_o^H \mathbf{W}_o \mathbf{w}_d,$$

Key Relations in * and our work

Research Gaps:

Assumes availability of $R/\widehat{R}$; not observable to HBFs

($\widehat{\boldsymbol{R}}_{\text{hyb}} \in \mathbb{C}^{P\times P}$; For the Phaser: PxP is 2x2, NxN is 8x8)

No prior real-world H-MVDR demo till date

xt Up: How do we overcome research gaps and get this working!

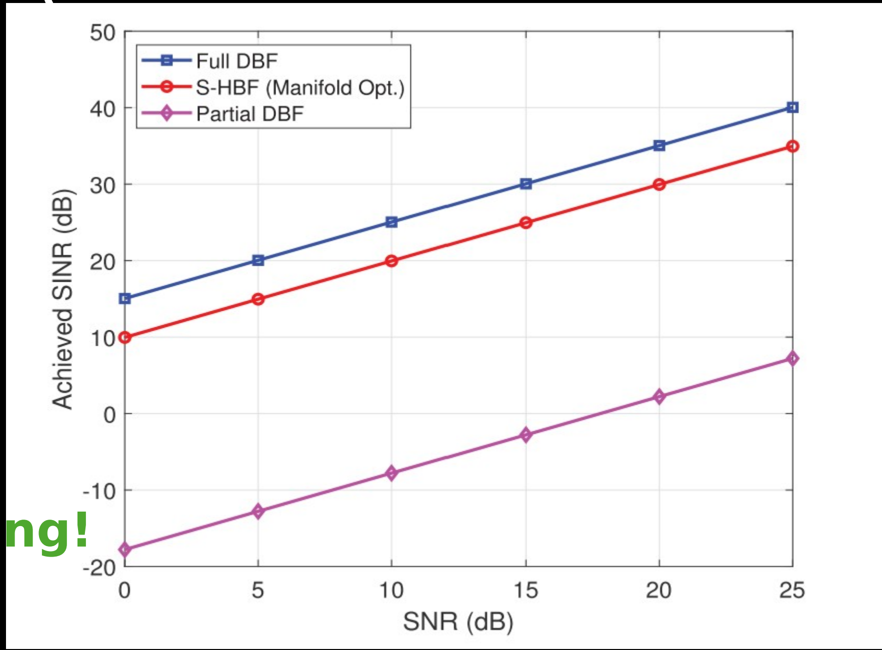


Figure: Chalise et. al*

$\mathbf{w}_o$ – Optimal MVDR weights, $\mathbf{W}_d/\mathbf{w}_d$ – Hybrid analog weights in matrix/vector forms, $\mathbf{w}$ – Hybrid digital weights, M – subarray size

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Overview of Approach

Collect and concatenate multiple $\widehat{R}_{hyb}$ across different Antenna Pairs (subsampling strategy)

Use collected $\widehat{R}_{hyb}$ estimates to construct incomplete estimate of completed sample covariance matrix $\widehat{R}_{est}$ and complete it using a two-step Matrix Completion (MC) process

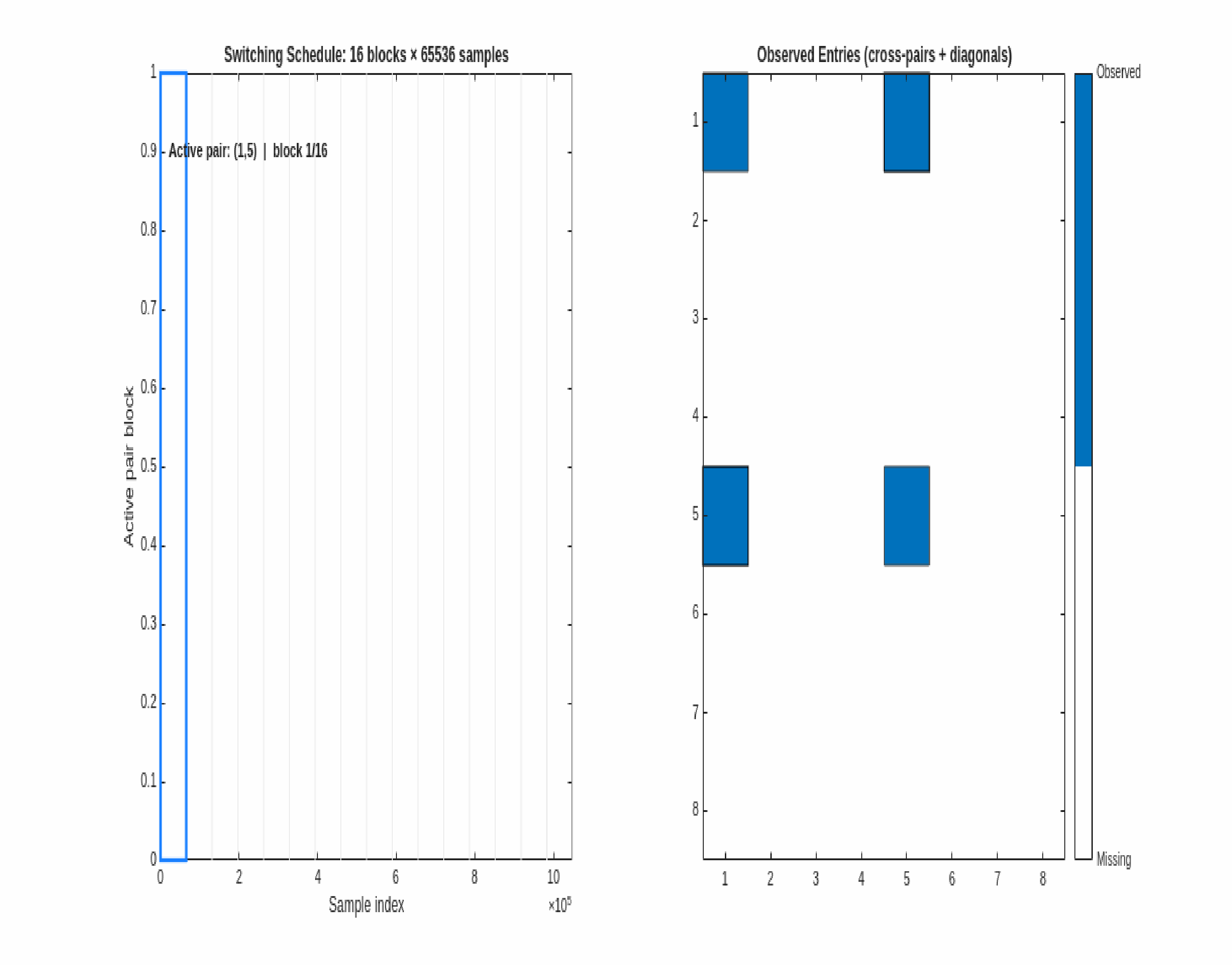
a. MC Initialization
b. MC Optimization

Using $\widehat{R}_{est}$ follow prior work's methodology to implement SMI for Hybrid

- Compare against F-DBF SMI, P-DBF SMI, P-HBF SMI (with known $\widehat{R}$)

Sampling Strategy

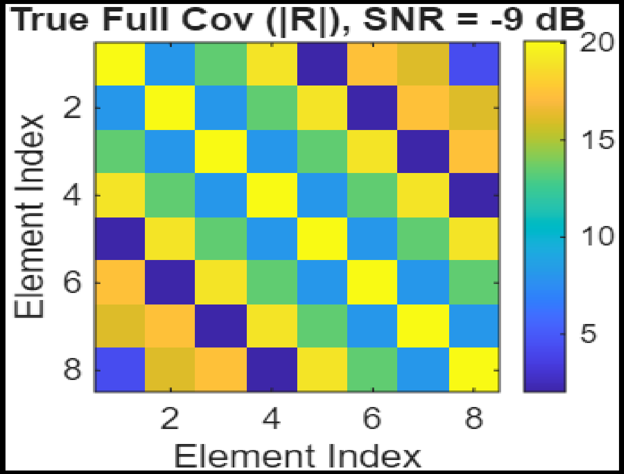
- Employ time-interleaved antenna multiplexing (TIAT) (as shown in the GIF on the right)
- Each subarray has one antenna active at a given time and its corresponding ADC acquires sample buffers (65k samples)
- Iterate through all possible pair combinations to acquire data across the array
- Primary limitation of this approach is that this still doesn't have access to all entries in the covariance matrix *
- Better of than when we started!! (from 4/64 to 40/64)



ondary Limitation- Assumes that the environment remains stationary across entire iterative acquisition loop

Matrix Completion - Initialization

- Ideal $\boldsymbol{R}$ matrices are Toeplitz- *only* N unique lags are needed to complete a N x N matrix
- $\widehat{\boldsymbol{R}}$ - for stationary processes are approximately Toeplitz
- For initialization, we leverage incomplete TIAT matrix and complete it using a Toeplitz Assumption



- Limitations:
  (i) Completed matrix not guaranteed Positive Semi-Definite (PSD); *vital* for SMI
  (ii) Naive Eigen Thresholding on Toeplitz completed $\widehat{\boldsymbol{R}}$ distorts and disregards collected measurements

Can we somehow get everything- Toeplitz, PSD, Numerical Stability, and Block Constraints (retain R values from TIAT) ?

Yes, we can!

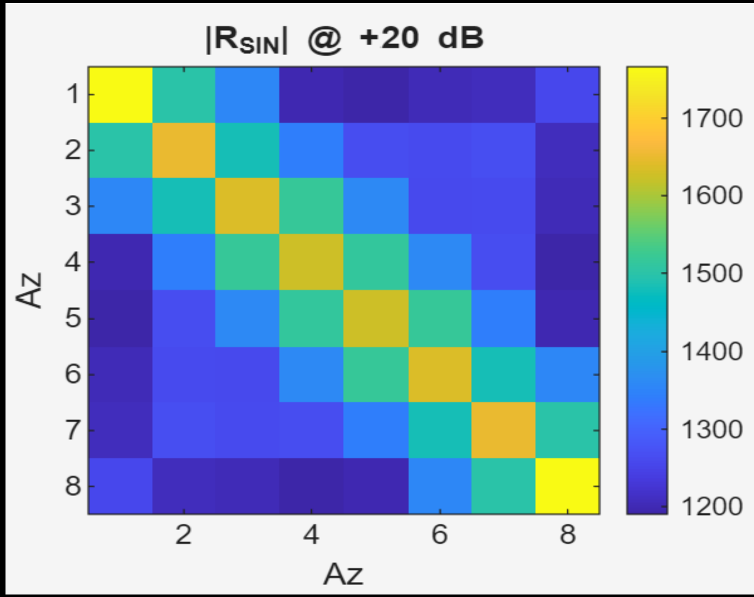


Fig: Measured Phaser $\widehat{\boldsymbol{R}}$

Matrix Completion - Optimization

We implemented Dykstra's Projection Algorithm with 3 constraints (PSD, Toeplitz, Block Constraints)

stored

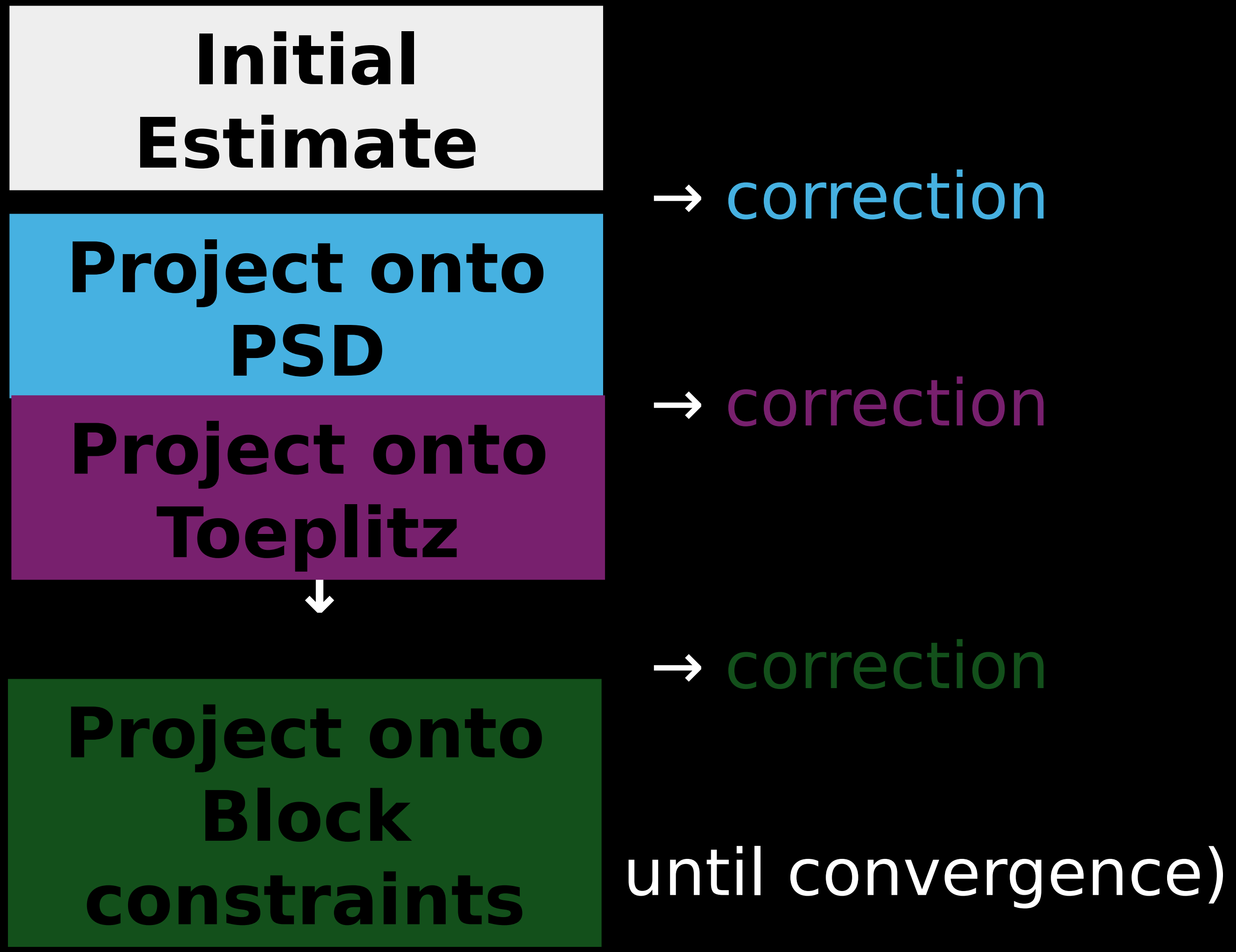
The Initial Estimate provided comes from the Toeplitz Completion Step covered Earlier

stored

The PSD step includes steps that transpose conjugate the matrix, eigen clip it towards PSD and a diagonal loading step that ensures numerical stability

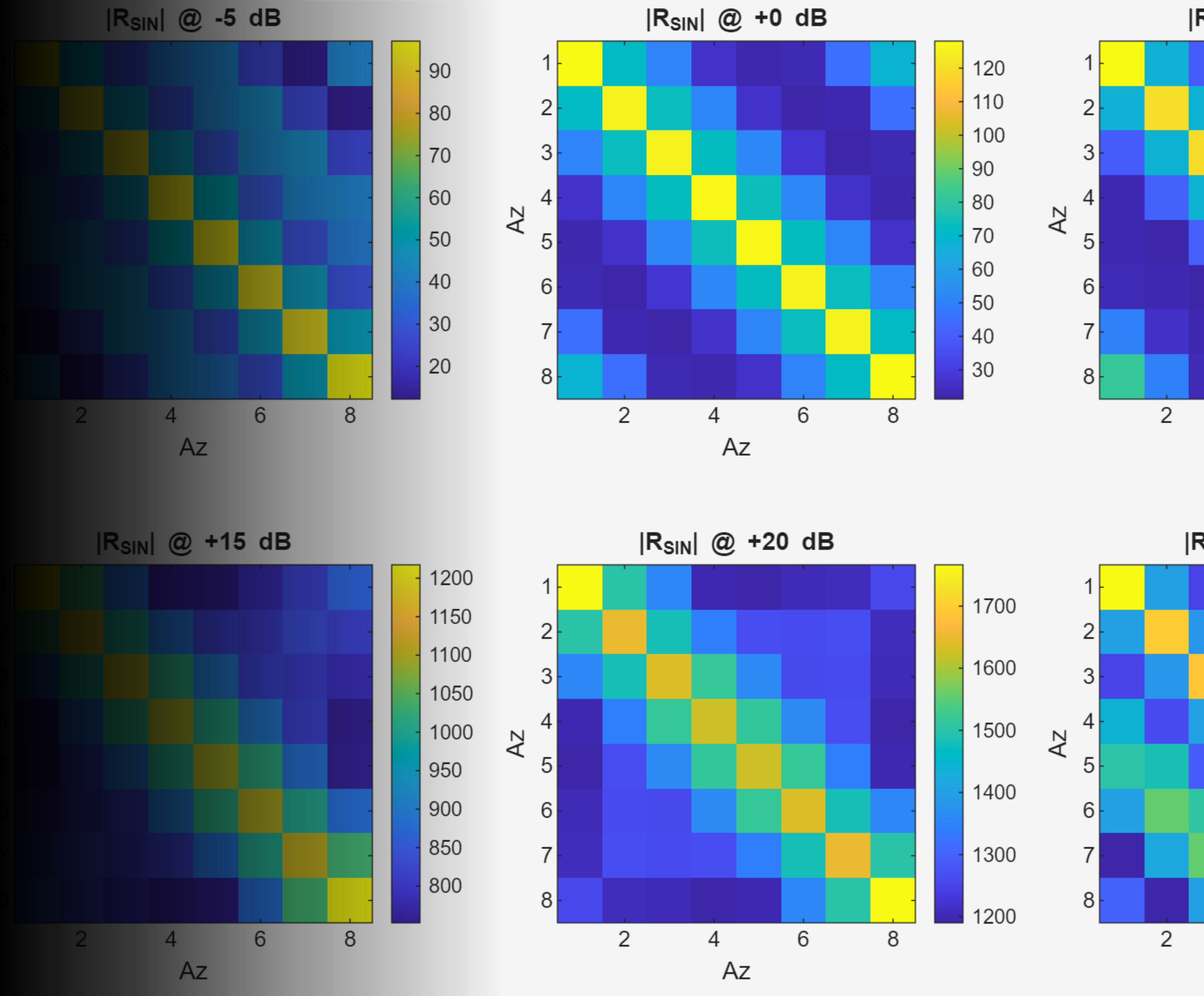
stored

Stored correction terms *guide* the iterates to converge to the nearest feasible solution in terms of Frobenius norm (even if no exact



Open Quiz! (Judges allowed to Participate)

- We don't see Toeplitz $\widehat{R}$ in practice…so why bother with it in the optimization loop?



Riemannian Manifold Optimization and Manopt

- The H-MVDR objective function is non-convex
- To optimize over this space efficiently we employ Riemannian Manifold Optimization
- Map hybrid digital and analog weights to a complex circle manifold of dimension N with unit modulus using ComplexCircleFactory
- Optimize for hybrid analog and digital weights using with a TrustRegions Optimizer
- Realize this on the discrete phase-shifters and the digital weights
- Implemented the optimization routine using Manopt on MATLAB

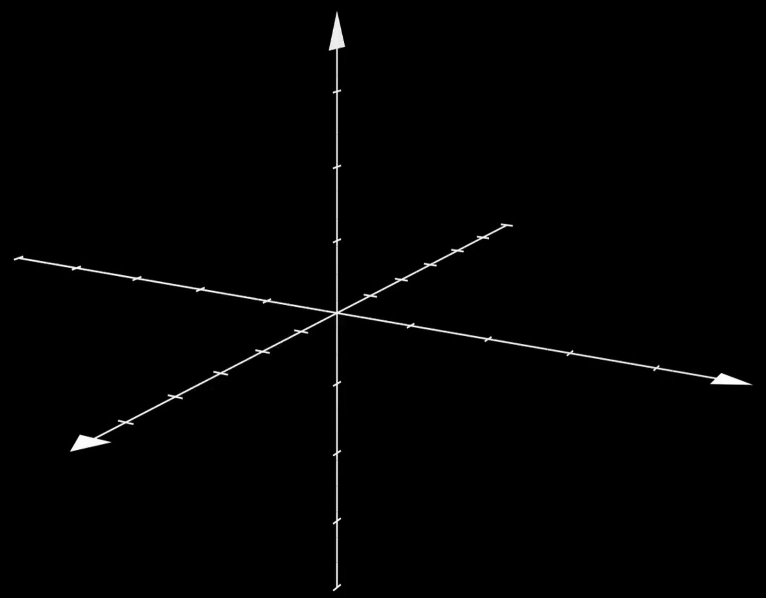


Figure: Visualization of a torus manifold representing two phase shifts*

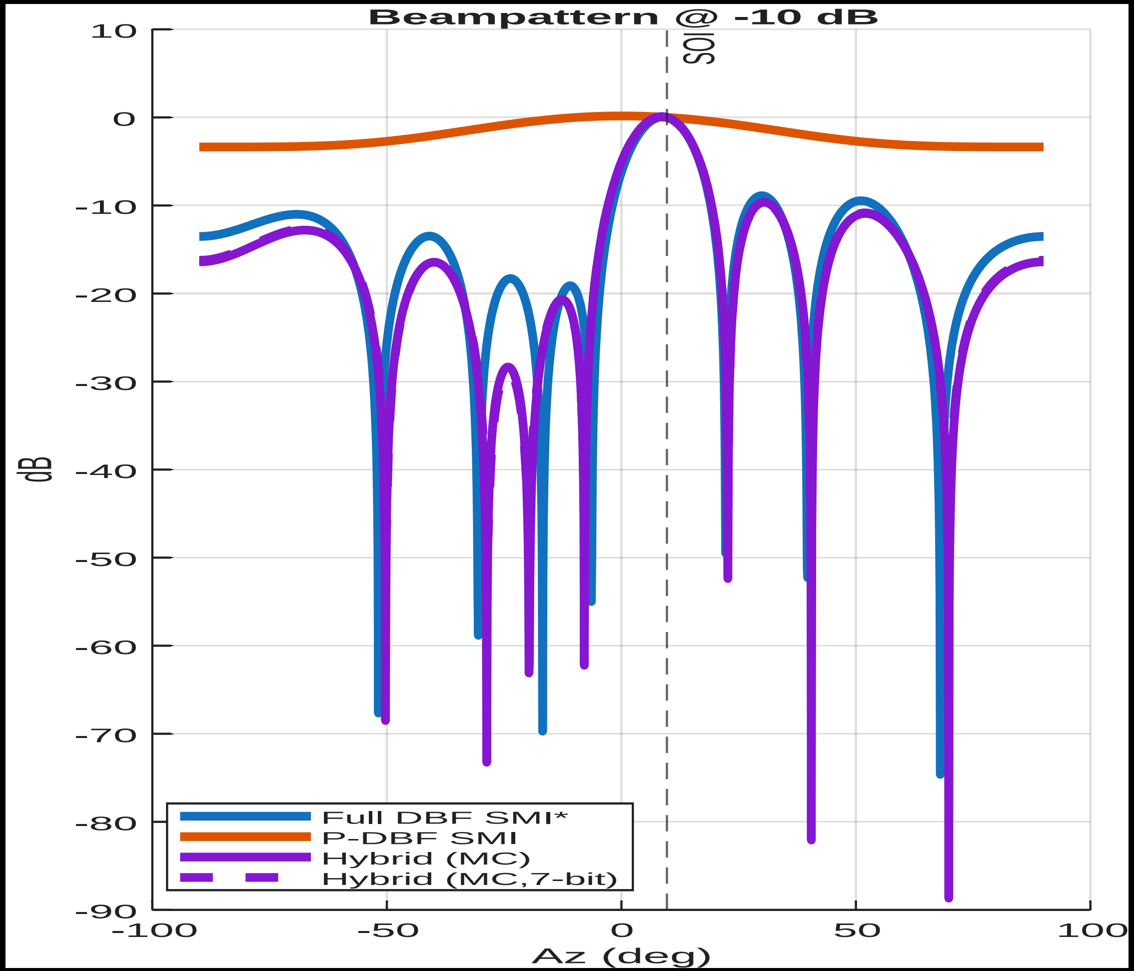
* Visualized using manim (https://github.com/3b1b/manim)

Phaser Setup

SoI , INT1 and INT2 are selected to fit in a 20 MHz BW. R&S SMW200A generates the SoI and INT1while HB100 generates INT2. INT2 distance and INT1 input power levels are calibrated till an INR of 20 dB is reached. SNR is varied between -10 to 20 dB to understand how H-SMI with MC **R** performs (recreating prior work).

SoI, INT1, INT2 AoA: ~ 10° ,- 20°, 40°

Results – Phaser

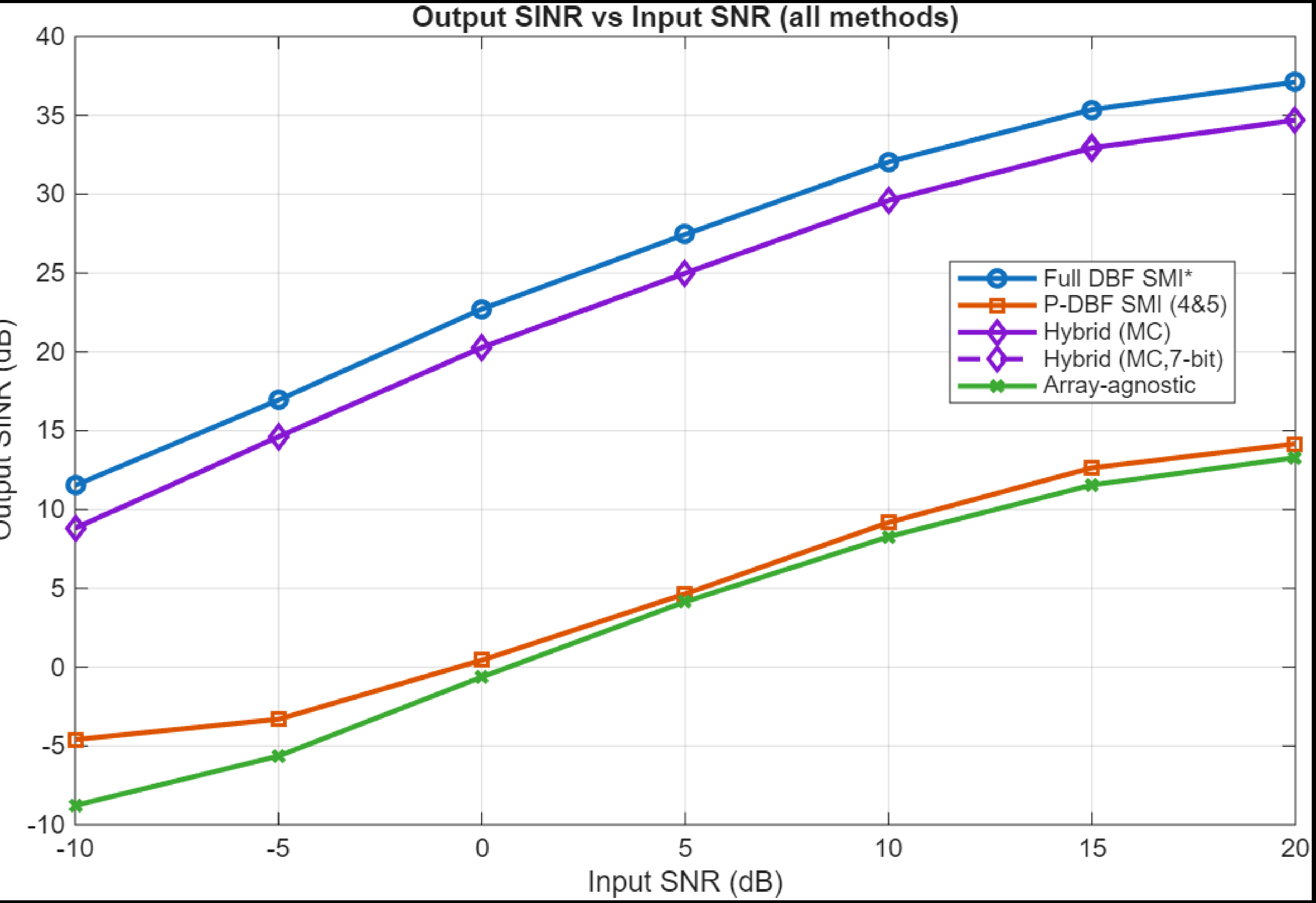


1. P-DBF SMI that only uses 2 elements struggles to null both interferers and maintain distortionless response at SOI

2. Weights obtained from MC techniques perform nearly identical with Full DBF SMI*

3. The full DBF SMI shown in the plot is slightly misleading- it represents how a Phaser with 8 dig channels would do perform if $\hat{\boldsymbol{R}}_{est}$ was $\hat{\boldsymbol{R}}$

Results – Phaser II



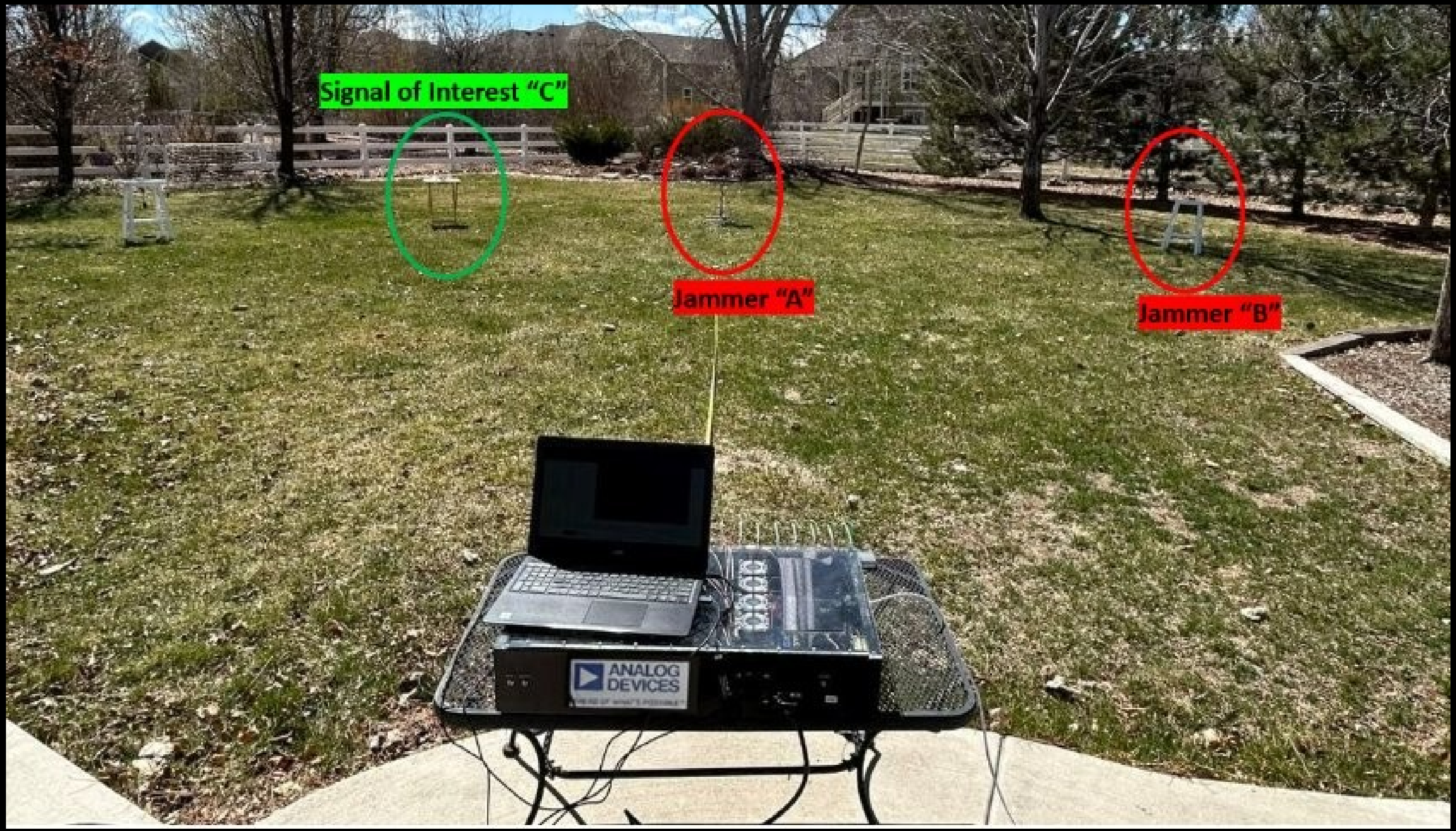
The P-DBF does slightly better than the SINR at every element when no BF is performed

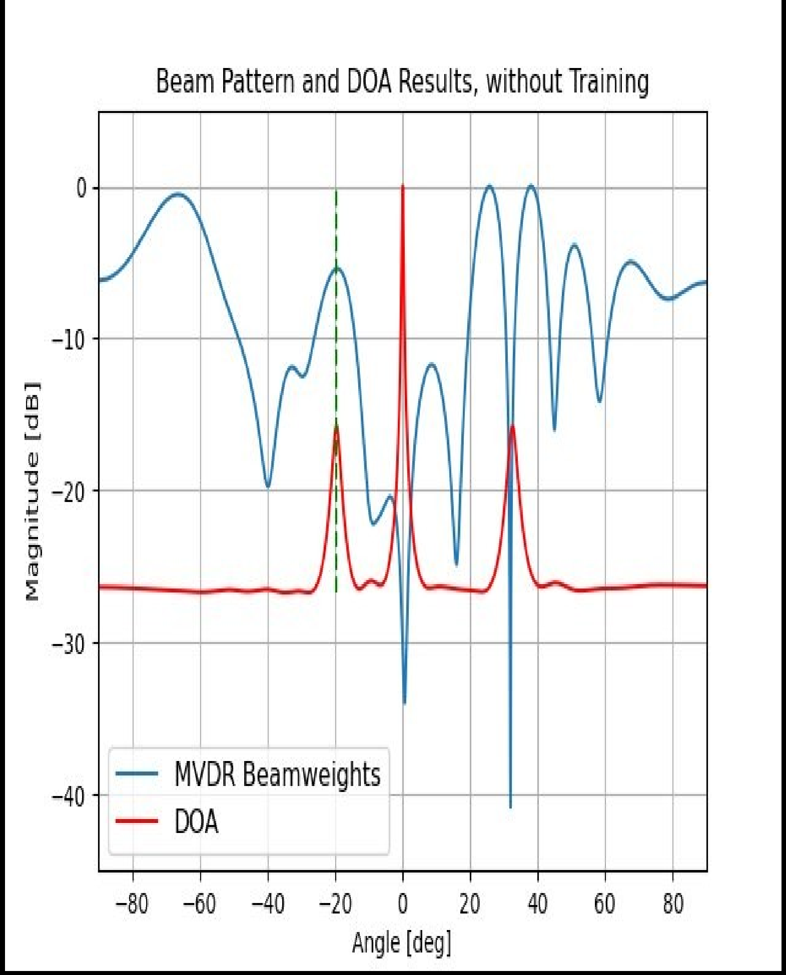
The MC Hybrid approach that we have developed performs 14-20 dB better than the P-DBF approach

The MC Hybrid approach lags the
full-DBF by only 3-4 dB and thus
presents optimistic results on the
viability of HBFs in DBF applications
where a 3-4dB performance

One last thing…

16 Channel MxFE Setup





MVDRLabs data collected by Jon Kraft, Analog Devices for Intl RadarCon 25.
Data consists of a SOI (~20°) and two interferers (~0°, ~35°)

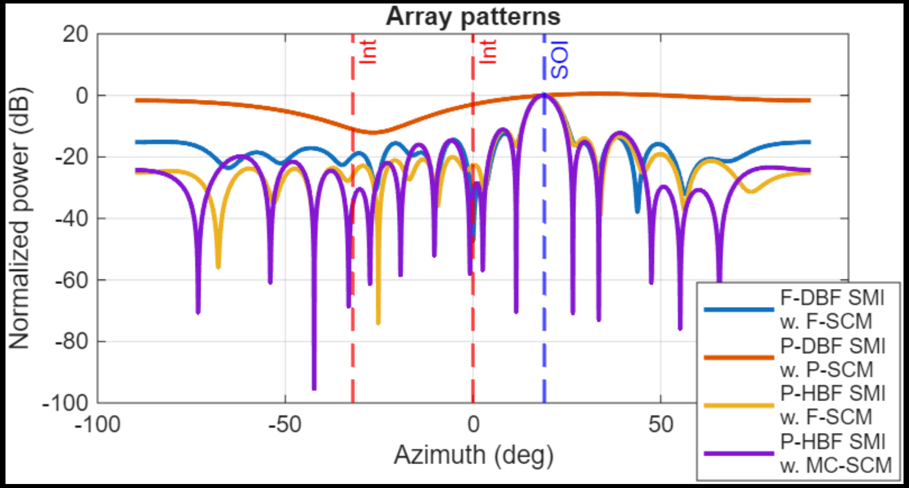
Results – 16 Channel MxFE

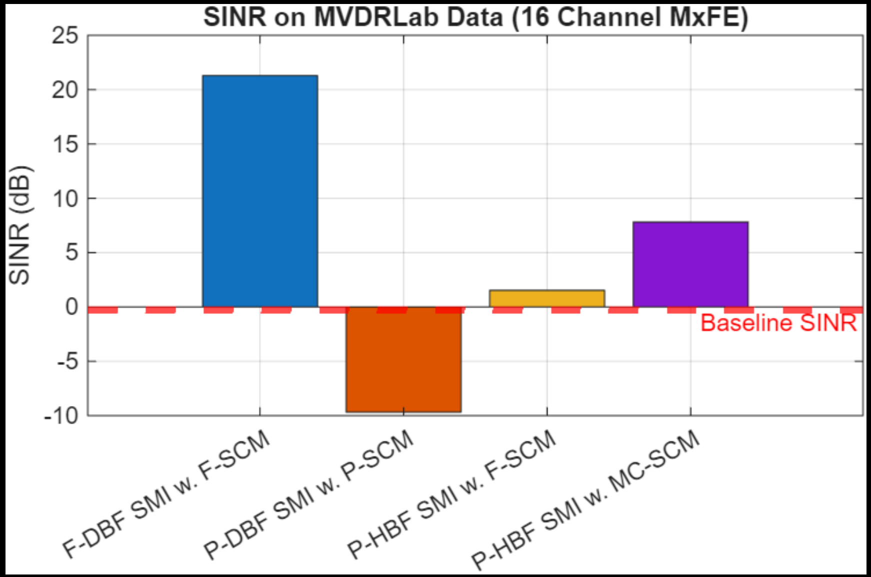
- Comparison of Patterns for Various Approaches – F-DBF (16 Dig Channel), P-DBF (2 Dig Channel + 2 Analog Channels), P-HBF w F-SCM (access to $\widehat{\boldsymbol{R}}$), P- HBF with MC $\widehat{\boldsymbol{R}}_{\text{est}}$

- We outperform the H-SMI that can access $\widehat{\boldsymbol{R}}$ by ~5 dB

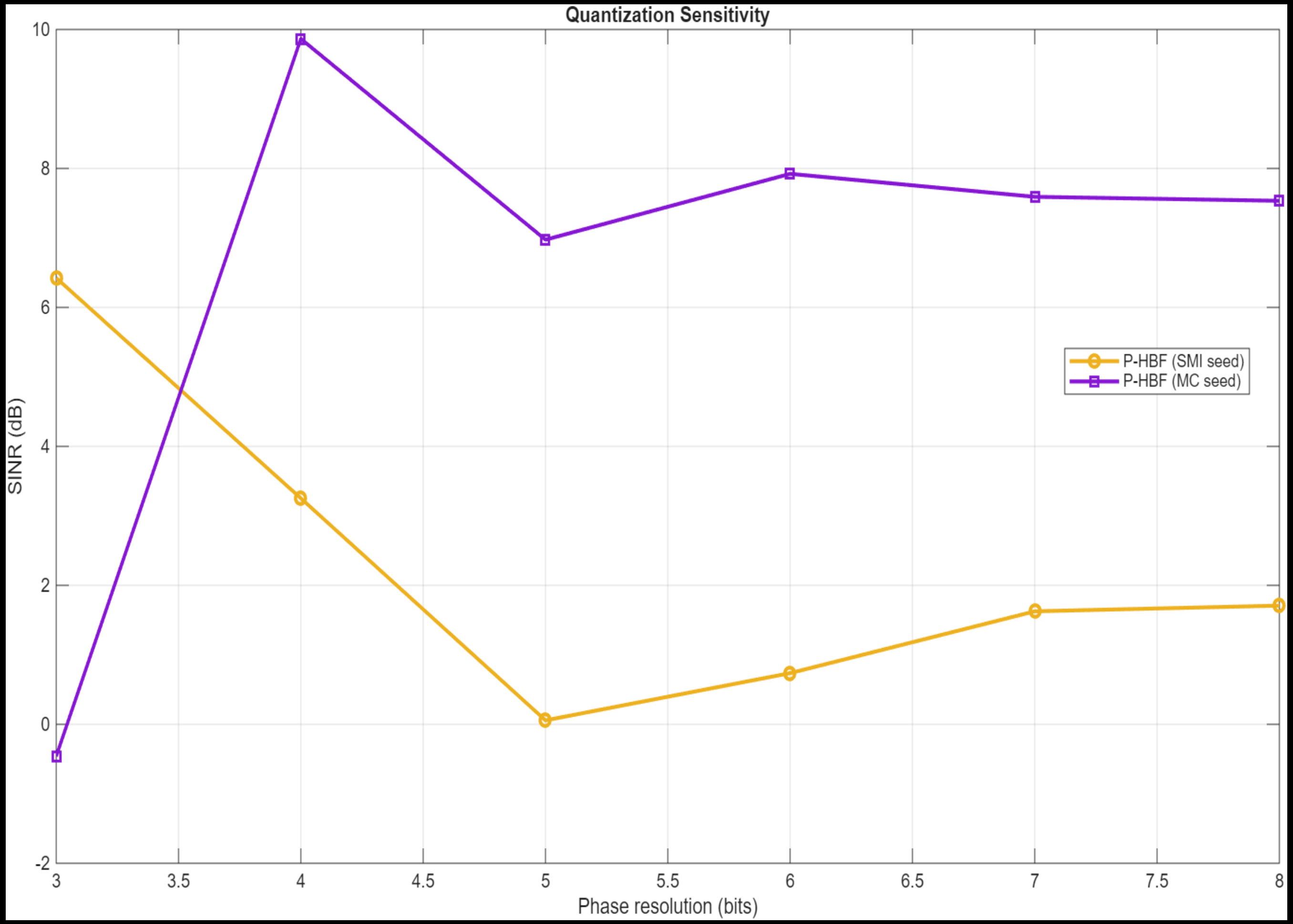
- We exceed baseline SINR by ~8 dB with 16 elements and 2 Dig channels

- In the 16 Channel case- our approach lags the Full-DBF approach by about 10-12 dB





Results – 16 Channel MxFE



We analyzed phase quantization from discrete phase shifters affected different hybrid strategies

Sometimes Phase shifters with lower resolution perform better

Current Thought:

- SINR’s sensitivity to nulls; higher phase resolutions result in deep nulls and slight offsets lead to large drops in SINR.
- Likewise, lower phase resolution are more robust to small offsets as they offer a shallower albeit graceful rolloff

Our Contributions

- Devised a recovery process for $\widehat{\boldsymbol{R}}_{\text{est}} \in \mathbb{C}^{N \times N}$, which is an estimate of $\widehat{\boldsymbol{R}}$ from $\widehat{\boldsymbol{R}}_{\text{hyb}} \in \mathbb{C}^{P \times P}$; this leads to realizability of prior work
- Investigated performance of different Manopt solvers and found better solvers (Trust regions and conjugate gradient)
- Implemented the world's first* Hybrid SMI (H-SMI) using the recovered $\widehat{\boldsymbol{R}}_{\text{est}}$
- Provide Comparisons: P-DBF SMI, F-DBF SMI, and HBF MVDR-SMI(prior work)
- Quantified effect of q-bit phase quantization in phase shifters as well as explored implications of array size

*to the best of our knowledge

Conclusion

- To wrap up — we developed a **structure-aware covariance completion method** for hybrid arrays, using **Dykstra's projections** across **Toeplitz**, **block-consistency**, and **PSD** constraints.
- We implemented and tested it on the **ADALM-Phaser** and **MxFE** platforms, and showed that our **hybrid SMI–MVDR** approach **outperforms prior hybrid MVDR baselines** — all **without needing the full, clairvoyant covariance matrix** that those prior methods assumed.

Current work assumes stationarity in the environment between

Thank You!

Any Questions?