Target Direction Finding and Localization with Phaseless Measurements

Dr Wei Liu

IEEE AESS Distinguished Lecturer

Antennas and Electromagnetics Research Group
School of Electronic Engineering and Computer Science
Queen Mary University of London, UK

Email: w.liu@qmul.ac.uk; wliu.eee@gmail.com
https://sites.google.com/site/liuweipage

Outline

1. Motivation for Magnitude-only/Phaseless Measurements

2. Associated Ambiguities for Direction of Arrival (DOA) Estimation

3. Group Sparsity Based Phase Retrieval for Direction Finding

4. Target Localization Based on Distributed Array Networks

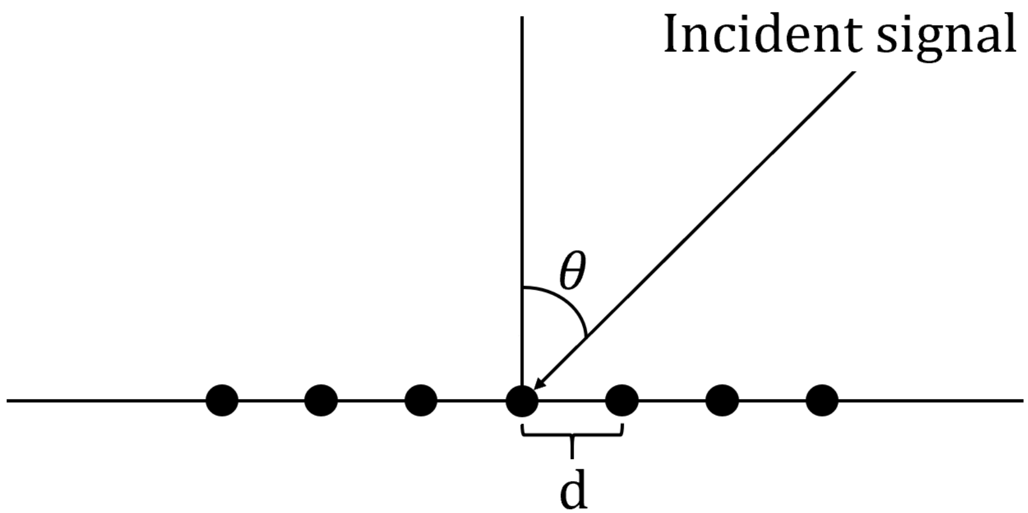
5. Conclusions

1. Motivation for Phaseless Measurements

- Target direction finding and localization is an important problem in radar detection and estimation based on antenna arrays.
- Normally a full measurement (including both phase and magnitude) of the received signals is needed for effective detection and estimation and many classic direction of arrival (DOA) estimation algorithms are based on this assumption, such as MUSIC and ESPRIT.
- However, this would need prior calibration of the whole array system to remove any possible phase errors present in the measurements, which may not be realistic in many practical scenarios and phase errors are not avoidable as a result.

1. Motivation for Phaseless Measurements

- To show the benefits of taking magnitude-only or phaseless measurements, consider the following N-sensor uniform linear array (ULA) as an example,



with distance d between adjacent sensors, and θ is measured from the broadside of the array.

1. Motivation for Phaseless Measurements

- With K incident signals, without model errors, the measurements at the p-th time instant is given by (noise-free):

$$\mathbf{x}[p] = \mathbf{A}(\theta)\mathbf{s}[p], \tag{1}$$

where $\mathbf{x}[p] = [x_1[p], ..., x_N[p]]^T$ and $\mathbf{s}[p] = [s_1[p], ..., s_K[p]]^T$ are the received signal and source signal vectors, respectively, and $\mathbf{A} = [\mathbf{a}_1, ..., \mathbf{a}_K]$ is the steering matrix of the array, with

$$\mathbf{a}_k = [1, e^{-j2\pi\frac{d}{\lambda}(\sin\theta_k)}, ..., e^{-j(N-1)2\pi\frac{d}{\lambda}(\sin\theta_k)}]^T. \tag{2}$$

- For P snapshots, one has

$$\mathbf{X} = \mathbf{A}(\theta)\mathbf{S}, \tag{3}$$

where $\mathbf{X} = [\mathbf{x}[1], ..., \mathbf{x}[P]]$ and $\mathbf{S} = [\mathbf{s}[1], ..., \mathbf{s}[P]]$.

1. Motivation for Phaseless Measurements

- Now suppose there is an unknown phase error e_n at sensor n of the array. Then,

$$\mathbf{X} = \mathbf{EAS}, \tag{4}$$

where $\mathbf{E}$ is a diagonal matrix with its diagonal given by e^{je_n}.

- e^{je_n} could be corrected by some array calibration techniques; however, it can be eliminated without calibration by simply taking the absolute value of each received signal, i.e.

$$\mathbf{Y} = |\mathbf{X}| = |\mathbf{EAS}| = |\mathbf{E}||\mathbf{AS}| = |\mathbf{AS}| \tag{5}$$

where $|\cdot|$ is the element-wise absolute value operator. So phase error has been removed by magnitude-only measurements.

- Or we could directly design a system which only measures signal magnitude and one good example is the Rydberg quantum sensor, which only measures the intensity of an impinging electric field.

1. Motivation for Phaseless Measurements

- Consider the more realistic case with uncorrelated additive noise

$$\mathbf{Y} = |\mathbf{AS}| + \mathbf{N}, \tag{6}$$

  where **N** is the noise matrix.

- A big question is, can we still perform effective DOA estimation?

- Actually this is a classic phase retrieval problem, where only (squared) signal magnitude is available. The only difference is, in traditional phase retrieval applications, such as optical imaging [1], there is usually only one snapshot, while in the above model, multiple snapshots are available.

- So group sparsity could be incorporated into existing phase retrieval algorithms to solve the problem; however, due to loss of phase information, there are inherent ambiguities for DOA estimation.

2. Associated Ambiguities for DOA Estimation

- One general observation is, if there is only one impinging signal, then there is no way to determine its direction as the magnitude of array measurement with a single impinging signal will stay the same for all N sensors, no matter which direction the signal comes from.

- To remove this ambiguity, we can simply put one arbitrary source to a direction of convenient choice to make sure there will be at least two signals present. Note that we do not need to know exactly which direction this additional source comes from.

- However, it is difficult to have a further more general discussion about ambiguities, as they are normally array structure dependent; next, we will use the popular ULA as an example for our discussion and introduce two array structures to tackle the issue.

- Three ambiguities are associated with a ULA: mirroring, spatial shift and spatial order ambiguities, as explained in the following [2, 3].

2. Associated Ambiguities for DOA Estimation

- Mirroring ambiguity refers to a mirrored version $\mathbf{s}^* = [s_1^*, ..., s_K^*]$ of the original signals from angle $-\theta_k$ would lead to the same magnitude measurements as the original ones.

- Denote $\check{x}_n$ as measurements of the mirrored signal

$$\begin{aligned} |x_n| = |x_n^*| &= |\sum_{k=1}^{K} s_k^* (e^{-jn\alpha \sin\theta_k})^*| \\ &= |\sum_{k=1}^{K} s_k^* e^{-jn\alpha \sin(-\theta_k)}| = |\check{x}_n| \end{aligned} \tag{7}$$

  where $'*'$ represents complex conjugate and $\alpha = 2\pi\frac{d}{\lambda}$.

- Basically due to the $\sin\theta$ term, this mirroring ambiguity will arise for any linear array structure.

2. Associated Ambiguities for DOA Estimation

- Spatial shift ambiguity refers to a spatially shifted version of the original source from angle θ_k by an amount ϕ/α would still have the same magnitude measurements as the original one.

- Denote $\check{x}_n$ as measurements of the shifted signal and $\ddot{\theta}_k$ the shifted DOA.

$$
\begin{aligned}
|x_n| &= |\sum_{k=1}^{K} s_k e^{-jn\alpha\sin\theta_k} e^{-jn\phi}| = |\sum_{k=1}^{K} s_k e^{-jn\alpha(\sin\theta_k + \frac{\phi}{\alpha})}| \\
&= |\sum_{k=1}^{K} s_k e^{-jn\alpha\sin\ddot{\theta}_k}| = |\check{x}_n|
\end{aligned} \tag{8}
$$

2. Associated Ambiguities for DOA Estimation

- Spatial shift ambiguity will not affect the DOA order, i.e. with $\sin\theta_1 < \sin\theta_2 < ... < \sin\theta_K$, we have $\sin\ddot{\theta}_1 < \sin\ddot{\theta}_2 < ... < \sin\ddot{\theta}_K$.

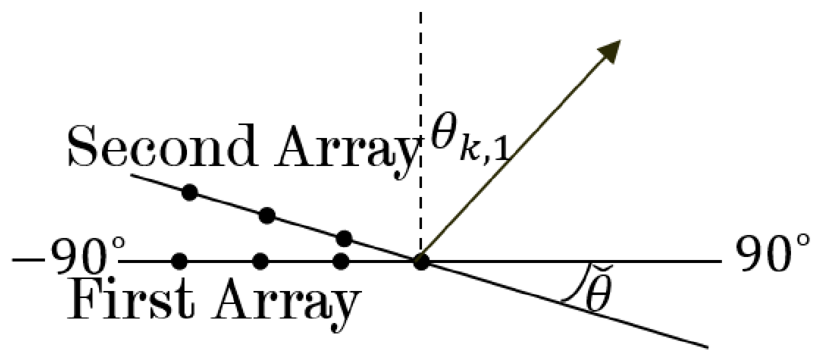
- However, there is another ambiguity and we call it "spatial order ambiguity", as this ambiguity will change the spatial order of the impinging signals, i.e. with $\sin\theta_1 < \sin\theta_2 < ... < \sin\theta_K$ we can not have $\sin\ddot{\theta}_1 < \sin\ddot{\theta}_2 < ... < \sin\ddot{\theta}_K$.

- This ambiguity occurs when applying half-wavelength spaced ULA to the full angle range $[-90°, 90°]$ and cannot be solved by placing known reference signal(s). But it can be avoided by limiting the adjacent sensor spacing to $\lambda/4$.

- The analysis for this ambiguity is complicated and omitted here. For detail, please refer to [4].

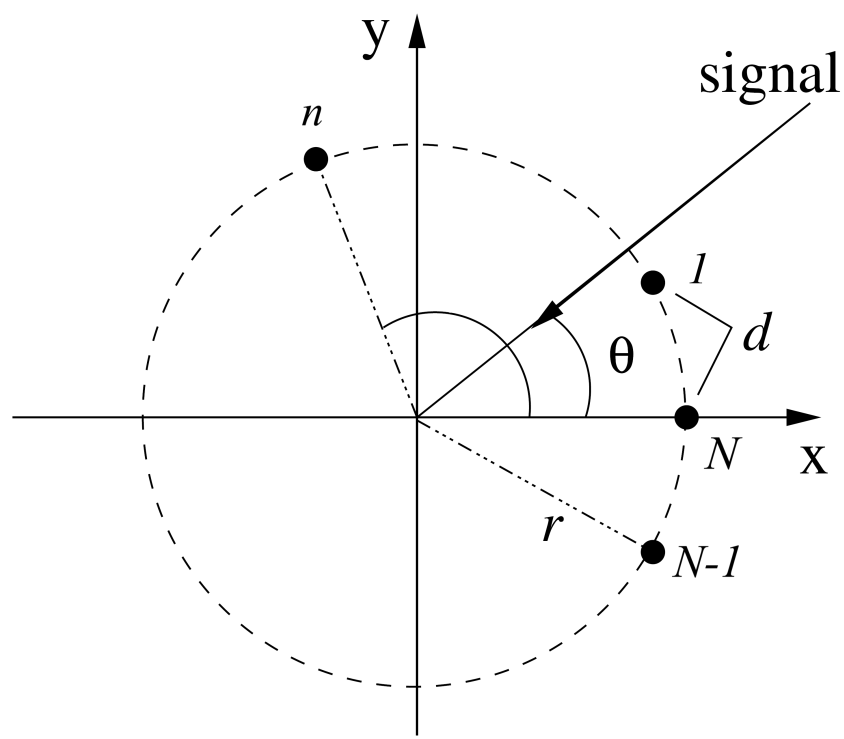
2. Associated Ambiguities for DOA Estimation

- To avoid the ambiguities associated with the ULA structure, one solution is to employ a dual array system [4, 5].
- It consists of two ULAs, where the first array is in the horizontal direction, while the second has a known angle $\check{\theta}$ to the first array.
- Due to the nonzero rotation angle, any shift or mirroring will lead to inconsistency in the two sets of estimated DOA results.



2. Associated Ambiguities for DOA Estimation

- Another choice is to employ the uniform circular array (UCA) [6].
- The UCA can be considered as multiple pairs of dual-arrays with two sensors for each subarray and therefore can overcome the ambiguities issue associated with a single ULA.



2. Associated Ambiguities for DOA Estimation

- For the UCA, its steering vector is given by

$$\mathbf{a}(\theta_k) \quad = [e^{j\xi\cos(\theta_k-\gamma_1)}, ..., e^{j\xi\cos(\theta_k-\gamma_N)}]^T, \tag{9}$$

where $\xi = 2\pi r/\lambda$, and $\gamma_n = 2\pi n/N$, $n = 1, ..., N$.

- For mirroring, signals arriving from $-\theta_k$ generate measurements with the same magnitude from the original ones $|x_n| = |\sum_{k=1}^{K} s_k e^{j\xi\cos(\theta_k-\gamma_n)}|$.

$$|\check{x}_n| = |\sum_{k=1}^{K} s_k^* e^{j\xi\cos(-\theta_k-\gamma_n)}| = |\sum_{k=1}^{K} s_k^* e^{j\xi\cos(\theta_k+\gamma_n)}| \neq |x_n|. \tag{10}$$

- As the magnitude of $\check{x}_n$ is in general different from x_n, mirroring ambiguity will not appear.

2. Associated Ambiguities for DOA Estimation

- For spatial shift, the received signals are phase shifted by a specific amount ϕ_n,

$$\check{x}_n = e^{j\xi\phi_n}\sum_{k=1}^{K} s_k e^{j\xi\cos(\theta_k-\gamma_n)} = \sum_{k=1}^{K} s_k e^{j\xi\cos(\theta_k-\check{\theta}_{n,k}-\gamma_n)}. \tag{11}$$

- Although $\check{x}_n$ would share the same magnitude as x_n at the n-th sensor, $\check{\theta}_{n,k}$ for the corresponding k-th signal at different sensors are different due to the non-linear property of the $\cos$ function and the involvement of γ_n.

- This implies that, there is no common shift variable ϕ_n to simultaneously keep the same magnitude as x_n and same shifted angle $\check{\theta}_{n,k}$ for all N sensors.

2. Associated Ambiguities for DOA Estimation

- But there is another ambiguity for UCAs. For the whole range $[-\pi, \pi]$, K incident signals **s*** from angle $(\theta_k \pm \pi)$ would share the same magnitude as x_n, expressed as

$$\check{x}_n = \sum_{k=1}^{K} s_k^* e^{j\xi \cos(\theta_k \pm \pi - \gamma_n)} = \sum_{k=1}^{K} s_k^* e^{-j\xi \cos(\theta_k - \gamma_n)} = x_n^*. \qquad (12)$$

- As a solution, we can limit the area of interest to $[-90^\circ, 90^\circ]$, since for $-\pi/2 \leq \theta_k \leq \pi/2$, $\theta_k \pm \pi$ will exceed the limit.

- Actually this ambiguity happens to all two-dimensional (2-D) arrays considering the full 360° range.

- The ULA would suffer from the same ambiguity, but it is ignored, as for a ULA, the interested angle range is $[-90^\circ, 90^\circ]$.

3. Group Sparsity Based Phase Retrieval for Direction Finding

- Now based on the magnitude-only (non-coherent) measurement data model as shown below, a fast group sparsity based phase retrieval algorithm is developed [4].

$$\mathbf{Y} = |\mathbf{X}| = |\mathbf{A}\mathbf{S}| + \mathbf{N}. \tag{13}$$

- First uniformly divide the whole angle range of interest into $G \gg N$ grid points, represented by angles θ_g, $g = 1, 2, ..., G$.

- Then, the array output under the sparse representation framework is given by

$$\begin{gathered} \mathbf{Y} = |\tilde{\mathbf{A}}\tilde{\mathbf{S}}| + \mathbf{N}, \\ \tilde{\mathbf{A}} = [\mathbf{a}(\theta_1), ..., \mathbf{a}(\theta_G)], \qquad \tilde{\mathbf{S}} = [\tilde{\mathbf{s}}[1], ..., \tilde{\mathbf{s}}[p]]. \end{gathered} \tag{14}$$

3. Group Sparsity Based Phase Retrieval for Direction Finding

- With the sparse representation, the non-coherent DOA estimation problem can be formulated as a group sparse phase retrieval problem

$$\min_{\tilde{\mathbf{S}}} \||\tilde{\mathbf{A}}\tilde{\mathbf{S}}| - \mathbf{Y}\|_F^2 + \gamma\|\tilde{\mathbf{S}}\|_{2,1}, \tag{15}$$

  where γ is a trade-off factor between the two terms.

- Note that $\||\tilde{\mathbf{A}}\tilde{\mathbf{S}}| - \mathbf{Y}\|_F^2$ is non-convex and the problem cannot be solved directly.

- By applying the PRIME technique column by column to (15) [7], this non-convex problem can be majorised by a surrogate convex function as

$$\min_{\tilde{\mathbf{S}}} \|\tilde{\mathbf{A}}\tilde{\mathbf{S}} - \mathbf{C}^q\|_F^2 + \gamma\|\tilde{\mathbf{S}}\|_{2,1} \quad \text{with} \quad \mathbf{C}^q = \mathbf{Y} \odot e^{j\arg(\tilde{\mathbf{A}}\tilde{\mathbf{S}}^q)}. \tag{16}$$

3. Group Sparsity Based Phase Retrieval for Direction Finding

- The above problem can be readily solved by the proximal gradient method [8, 9]; at the $(q+1)$th iteration, $\tilde{\mathbf{S}}^{q+1}$ is given by (μ is stepsize)

$$\tilde{\mathbf{S}}^{q+1} = \underset{\mathbf{Z}}{\operatorname{argmin}}\{\|\frac{1}{2\mu}\|\mathbf{Z} - (\tilde{\mathbf{S}}^{q} - \mu\nabla F(\tilde{\mathbf{S}}^{q}))\|_F^2 + \gamma\|\mathbf{Z}\|_{2,1}\}. \tag{17}$$

- It has an analytical solution as [9, 10]

$$\tilde{\mathbf{s}}_i^{q+1} = (\tilde{\mathbf{s}}_i^{q} - \mu\nabla F(\tilde{\mathbf{s}}_i^{q}))\max(1 - \frac{\gamma\mu}{\|\tilde{\mathbf{s}}_i^{q} - \mu\nabla F(\tilde{\mathbf{s}}_i^{q})\|_2}, 0), \tag{18}$$

where $\tilde{\mathbf{s}}_i$ represents the i-th row of $\tilde{\mathbf{S}}$ and $\nabla F(\tilde{\mathbf{s}}_i^q) = 2(\tilde{\mathbf{A}}^H)_i(\tilde{\mathbf{A}}\tilde{\mathbf{S}}^q - \mathbf{C}^q)$, $i = 1, \cdots, G$, is the i-th row of $\nabla F(\tilde{\mathbf{S}}^q)$, with $(\tilde{\mathbf{A}}^H)_i$ being the i-th row of $\tilde{\mathbf{A}}^H$.

3. Group Sparsity Based Phase Retrieval for Direction Finding

- Nesterov acceleration can be applied to increase the converge speed [11].

- This method does not apply proximal operator to $\tilde{\mathbf{S}}^{q+1}$ directly, but another point $\mathbf{B}^{q+1}$ based on $\tilde{\mathbf{S}}^{q+1}$ and $\tilde{\mathbf{S}}^{q}$ expressed as

$$\mathbf{B}^{q+1} = \tilde{\mathbf{S}}^{q+1} + \frac{\beta^q - 1}{\beta^{q+1}}(\tilde{\mathbf{S}}^{q+1} - \tilde{\mathbf{S}}^{q}), \tag{19}$$

where

$$\beta^{q+1} = \frac{1 + \sqrt{1 + 4(\beta^q)^2}}{2}. \tag{20}$$

- The proposed algorithm is referred to as fasT grOup sparsitY Based phAse Retrieval (ToyBar) [4].

3. Group Sparsity Based Phase Retrieval for Direction Finding

Algorithm Summary (ToyBar)

Input: $\tilde{\mathbf{A}}$, $\mathbf{Y}$, γ, μ,

Output: $\tilde{\mathbf{S}}$ (reconstructed signal).

Initialization: Set $\tilde{\mathbf{S}}^0$ as a random matrix, $\mathbf{B}^0 = \tilde{\mathbf{S}}^0$, $\beta^0 = 1$.

General steps: for q=0, ..., Q

1) Calculate $\mathbf{C}^q = \mathbf{Y} \odot e^{j\arg(\tilde{\mathbf{A}}\mathbf{B}^q)}$

2) Calculate $\tilde{\mathbf{S}}^{q+1}$, for i=1, ..., G

With $\nabla F(\tilde{\mathbf{b}}_i^q) = 2(\tilde{\mathbf{A}}^H)_i(\tilde{\mathbf{A}}\tilde{\mathbf{B}}^q - \mathbf{C}^q)$, find $\tilde{\mathbf{s}}_i^{q+1}$ as

$\tilde{\mathbf{s}}_i^{q+1} = (\mathbf{b}_i^q - \mu\nabla F(\mathbf{b}_i^q))\max(1 - \frac{\gamma\mu}{\|\mathbf{b}_i^q - \mu\nabla F(\mathbf{b}_i^q)\|_2}, 0)$,

where $\mathbf{b}_i^q$ is the i-th row of $\mathbf{B}^q$.

3) **Update**: $\beta^{q+1} = \frac{1+\sqrt{1+4(\beta^q)^2}}{2}$,

$\mathbf{B}^{q+1} = \tilde{\mathbf{S}}^{q+1} + \frac{\beta^q - 1}{\beta^{q+1}}(\tilde{\mathbf{S}}^{q+1} - \tilde{\mathbf{S}}^q)$.

4) q=q+1, go to 1).

3. Group Sparsity Based Phase Retrieval for Direction Finding – Simulation Results

- Performance of the proposed ToyBar is compared with the modified GESPAR [2] based on the dual-array structure. For the modified GESPAR, 64000 iterations are used.

- For ToyBar, the iterations are fixed at $Q = 400$ and 50 random initializations are used to find the global minimum. Stepsize μ is set as $1/(2\lambda_{max}(\tilde{\mathbf{A}}^H\tilde{\mathbf{A}}))$.

- The angle between the two subarrys is $\check{\theta} = 20°$. A step size of $0.5°$ is used for initial DOA estimation.

- After obtaining the initial DOA estimates $\hat{\theta}$, a new grid with stepsize $0.05°$ is formed around $\hat{\theta}$, including $1.5°$ to each side.

- When applying the refining step to GESPAR, the number of iterations is halved as the number of grid points decreased.

3. Group Sparsity Based Phase Retrieval for Direction Finding – Simulation Results

- SNR is 15 dB; $K = 3$ signals with incident angles $-30°, -10°$, and $50°$ (relative to the first array); number of snapshots is 20, and number of sensors $N_1 = N_2 = 20$. Note that GESPAR requires prior knowledge of the number of incident signals, while ToyBar does not.

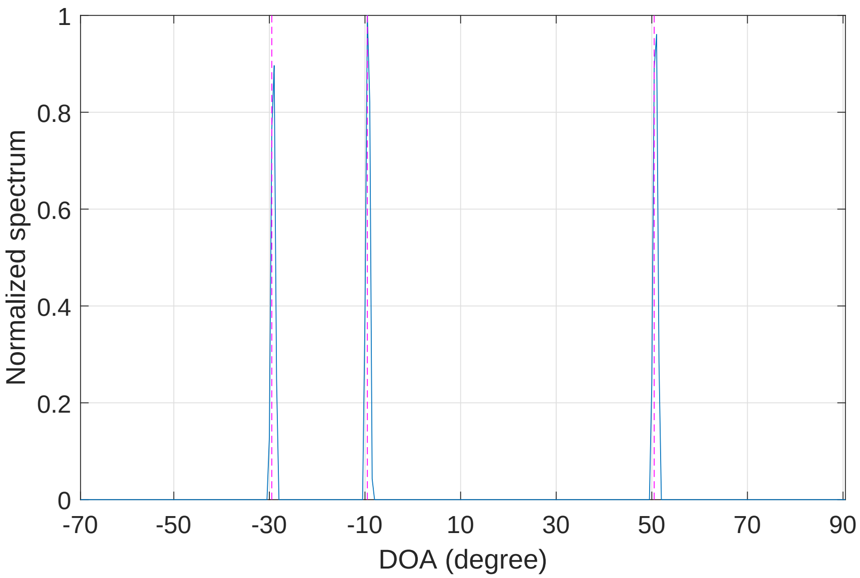


Fig. 1. Results by ToyBar.

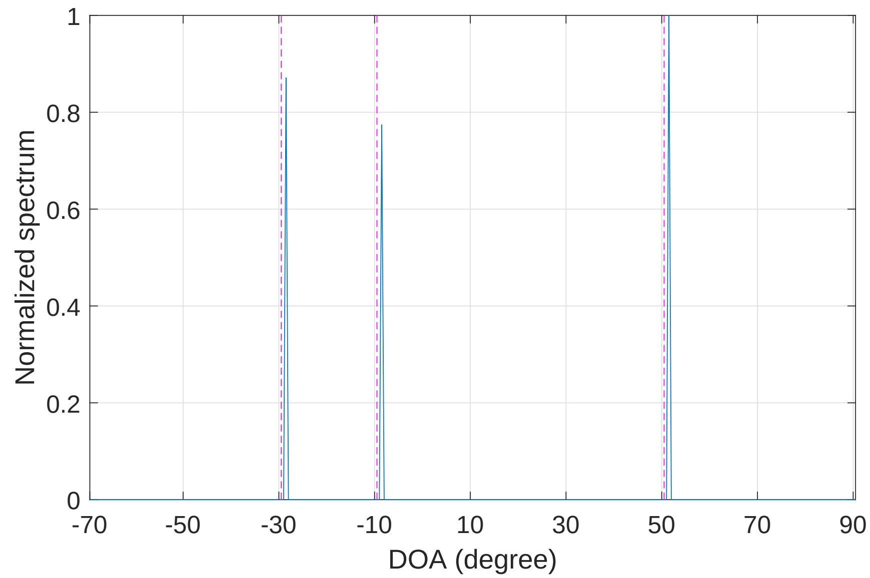


Fig. 2. Results by GESPAR.

3. Group Sparsity Based Phase Retrieval for Direction Finding – Simulation Results

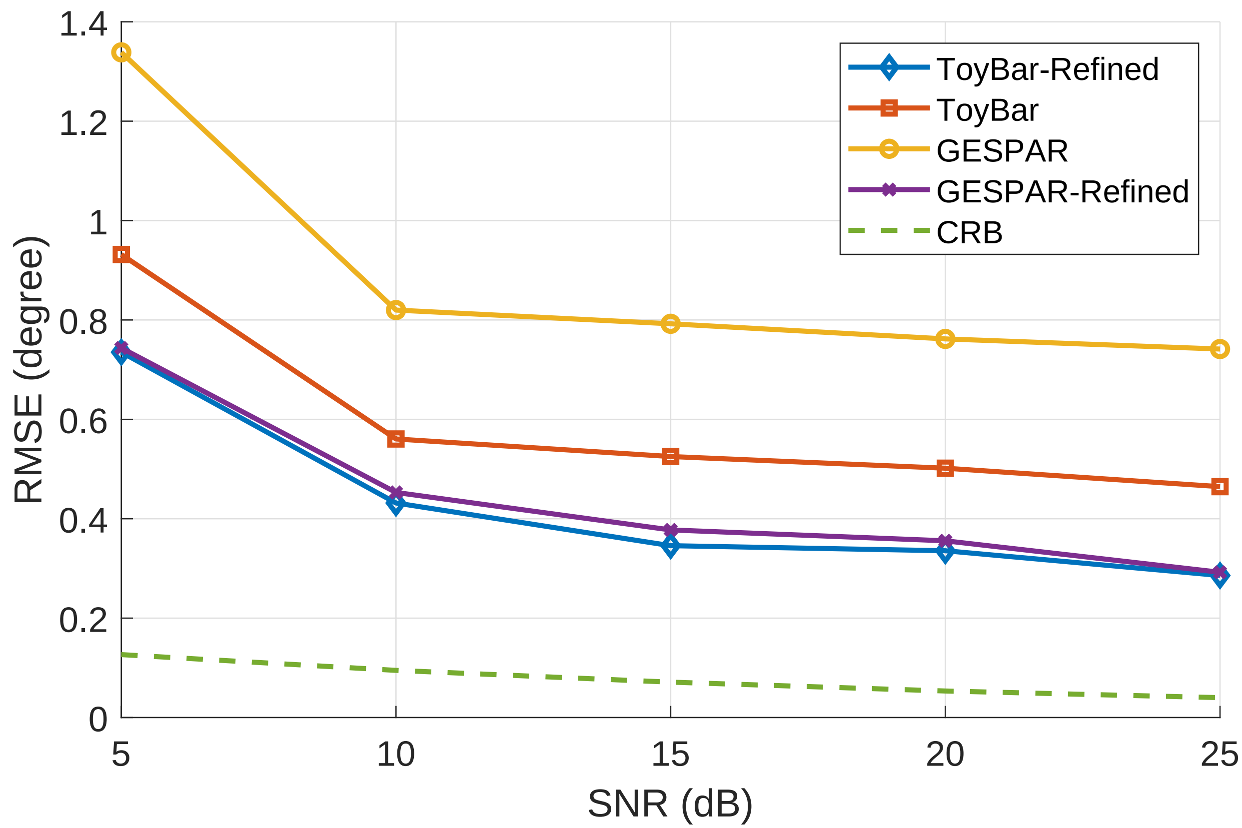


Fig. 3. RMSEs versus different SNR with 20 snapshots.

3. Group Sparsity Based Phase Retrieval for Direction Finding – Simulation Results

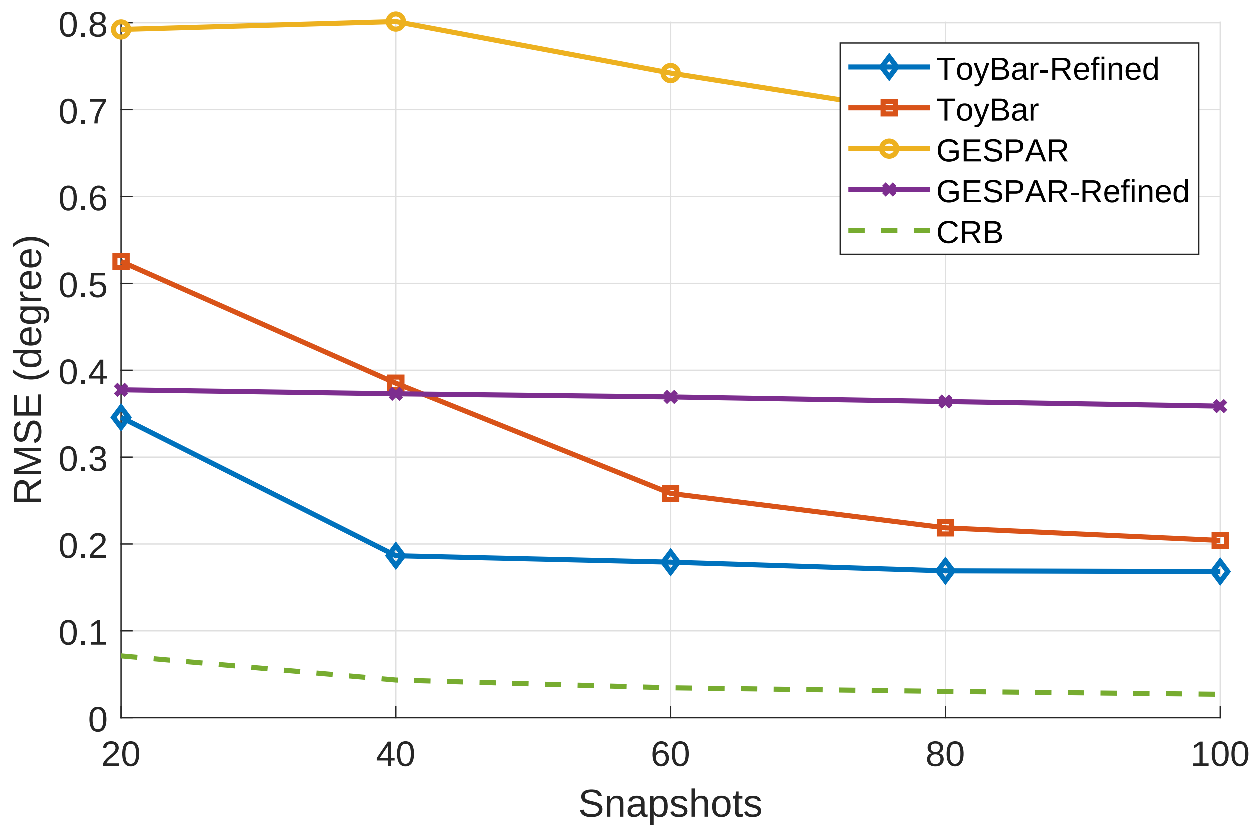


Fig. 4. RMSEs versus number of snapshots with SNR=15dB.

3. Group Sparsity Based Phase Retrieval for Direction Finding – Simulation Results

- Running time based on a PC with CPU I5 5200U at 2.2GHz and 4 GB RAM.

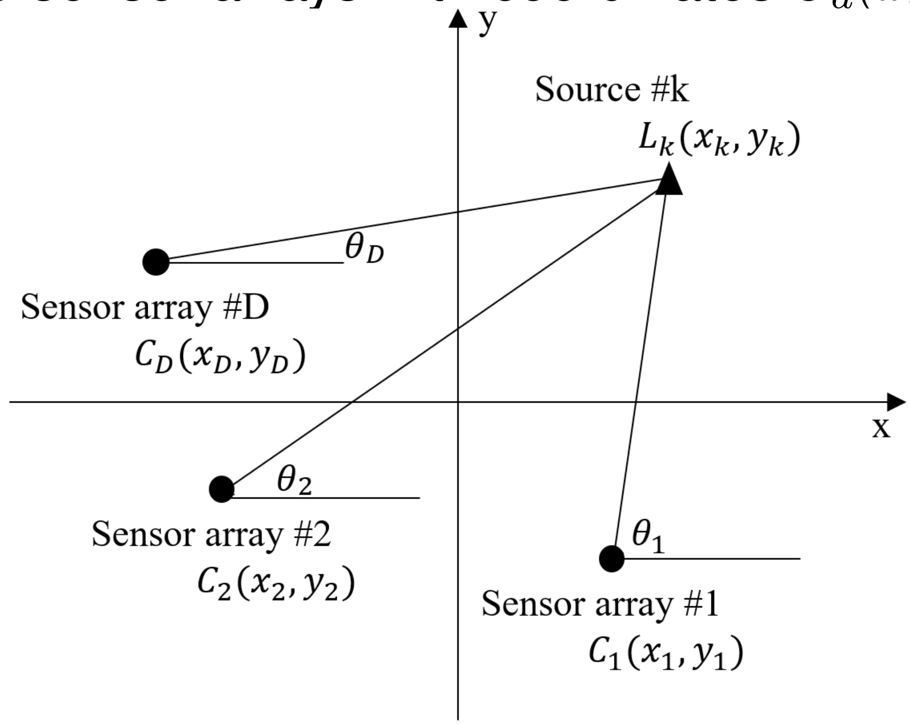
Table 1: Running times versus number of snapshots.

Snapshots	20	60	100
Toybar(s)	57.7	84.6	115.2
ToyBar-Refined(s)	89.2	134.1	181.4
GESPAR(s)	2154.2	6551.4	10816.5
GESPAR-Refined(s)	3384.7	10317.6	17045.5

4. Target Localization Based on Distributed Array Networks

- Source/target localization based on a network of distributed sensor arrays is a very important problem in array signal processing.

- Consider K narrowband sources s_k at locations $L_k(x_k, y_k)$ impinging on D deployed sensor arrays with coordinates $C_d(x_d, y_d)$.



4. Target Localization Based on Distributed Array Networks

- The number of sensors of the d-th array is N_d, and the corresponding magnitude-only measurements at the d-th array is expressed as

$$\mathbf{z}_d[p] = |\mathbf{A}_d \mathbf{s}_d[p]| + \mathbf{n}_d, \tag{21}$$

with $\mathbf{s}_d[p] = [s_{d,1}[p], ..., s_{d,K}[p]]^T$, where $s_{d,k}[p]$ represents the p-th snapshot of the k-th source signal corresponding to the d-th sensor array, $\mathbf{n}_d$ is the $N_d \times 1$ random Gaussian noise vector.

Note that here the source signal $s_{d,k}[p]$ can be independent of d so that all arrays receive the same group of source signals, but more generally, they can be different for different arrays.

$\mathbf{A}_d$ is the steering matrix with its columns $\mathbf{a}_d(\theta_{d,k})$, $k = 1, ..., K$, being the corresponding steering vectors

$$\mathbf{A}_d = [\mathbf{a}_d(\theta_{d,1}), ..., \mathbf{a}_d(\theta_{d,K})]^T. \tag{22}$$

4. Target Localization Based on Distributed Array Networks

- When employing a uniform circular array, $\mathbf{a}_d(\theta_{d,k})$ is given by

$$\begin{aligned} \mathbf{a}_d(\theta_{d,k}) &= [e^{j\frac{2\pi r}{\lambda}\cos(\theta_{d,k}-\gamma_1)}, ..., e^{j\frac{2\pi r}{\lambda}\cos(\theta_{d,k}-\gamma_{N_d})}], \\ \gamma_n &= 2\pi n/M_d, \end{aligned} \tag{23}$$

where λ is the signal wavelength and r the radius of the circular array.

- $\theta_{d,k}$ denotes the arriving angle between the k-th source and d-th sensor array, expressed as

$$\begin{aligned} \theta_{d,k} &= \text{arctan2}(\Delta y_{d,k}, \Delta x_{d,k}), \\ \Delta y_{d,k} &= y_k - y_d, \qquad \Delta x_{d,k} = x_k - x_d. \end{aligned} \tag{24}$$

with $\text{arctan2}(\cdot)$ being the inverse tangent operator.

4. Target Localization Based on Distributed Array Networks

- Collecting P snapshots to form $\mathbf{Z}_d = \big[\mathbf{z}_d[1], ..., \mathbf{z}_d[P]\big]$, one has

$$\begin{aligned} &\mathbf{Z}_d = |\mathbf{A}_d \mathbf{S}_d| + \mathbf{N}_d, \qquad \mathbf{S}_d = \big[\mathbf{s}_d[1], ..., \mathbf{s}_d[P]\big], \\ &\mathbf{N}_d = \big[\mathbf{n}_d[1], ..., \mathbf{n}_d[P]\big]. \end{aligned} \tag{25}$$

- Now we could apply the earlier proposed direction finding method directly to find all the DOAs relative to each array and then obtain target locations by finding intersections of those estimated DOAs.

- Disadvantages for the above solution: Firstly, since information at different observers is processed separately to obtain the individual DOAs, it is sensitive to estimation accuracy at each array, and one single unreliable estimate can cause large errors in the location results; secondly, there are also possible pairing and ambiguity problems associated with such a two-step approach.

4. Target Localization Based on Distributed Array Networks

- A more effective approach is to jointly exploit information across all sensor arrays by enforcing a common spatial sparsity as all target signals originate from the same set of target locations [12].

- Divide the admissible area of interest into G_x and G_y grid points along the x-axis and y-axis, separately ($G = G_xG_y$). The overcomplete steering matrix of the d-th sensor array can be expressed as

$$\tilde{\mathbf{A}}_d = [\mathbf{a}_d(\theta_{d,1}), \mathbf{a}_d(\theta_{d,2}), \cdots, \mathbf{a}_d(\theta_{d,G})], \tag{26}$$

  where $\theta_{d,g}$, $g = 1, 2, \cdots, G$, is the angle between the g-th grid point location (g_x, g_y) and the d-th sensor array,

$$\begin{aligned} \theta_{d,g} &= \operatorname{arctan2}(\Delta y_{d,g}, \Delta x_{d,g}), \\ \Delta y_{d,g} &= g_y - y_d, \qquad \Delta x_{d,g} = g_x - x_d. \end{aligned} \tag{27}$$

4. Target Localization Based on Distributed Array Networks

- Then, the sparse representation model for each array is

$$\mathbf{Z}_d = |\tilde{\mathbf{A}}_d \tilde{\mathbf{S}}_d| + \mathbf{N}_d. \tag{28}$$

- Note that incident sources from an arbitrary grid point would share the same spatial support in $\tilde{\mathbf{S}}_d$ and $\tilde{\mathbf{A}}_d$, $d = 1, ..., D$, although the arriving angles with respect to different arrays are different.

- Define a new steering matrix covering all D sensor arrays as

$$\tilde{\mathbf{A}} = \mathrm{blkdiag}\{\tilde{\mathbf{A}}_1, ..., \tilde{\mathbf{A}}_D\}. \tag{29}$$

Also define

$$\mathbf{Z} = [\mathbf{Z}_1^T, ..., \mathbf{Z}_D^T]^T,\ \tilde{\mathbf{S}} = [\tilde{\mathbf{S}}_1^T, ..., \tilde{\mathbf{S}}_D^T]^T,\ \mathbf{N} = [\mathbf{N}_1^T, ..., \mathbf{N}_D^T]^T. \tag{30}$$

4. Target Localization Based on Distributed Array Networks

- Now we reach the following overall model

$$\mathbf{Z} = |\tilde{\mathbf{A}}\tilde{\mathbf{S}}| + \mathbf{N}. \tag{31}$$

- Them, the source localization problem can be formulated as a joint group sparsity based optimization problem as follows [13].

$$\min_{\tilde{\mathbf{S}}} \|\mathbf{Z} - |\tilde{\mathbf{A}}\tilde{\mathbf{S}}|\|_F^2 + \gamma\|\hat{\mathbf{S}}\|_{2,1}, \text{ with } \hat{\mathbf{S}} = [\tilde{\mathbf{S}}_1, ..., \tilde{\mathbf{S}}_D]. \tag{32}$$

- The problem can be solved by the earlier proposed group sparsity based phase retrieval algorithm ToyBar.

- Similarly, we can employ grid refinement to reduce complexity of the group sparsity based solution, or we can develop an off-grid solution by employing Taylor series expansion [14].

4. Target Localization Based on Distributed Array Networks – Simulation Results

- Performance of the on-grid, grid-refinement, off-grid solutions is compared with the existing full-measurement on-grid method [12].

- There are $D = 4$ distributed arrays at $C_1 = (10, 40)$m, $C_2 = (30, 10)$m, $C_3 = (-80, 90)$m and $C_4 = (-20, 40)$m, while the off-grid locations for $K = 2$ sources are $L_1 = (-10.5, -9.5)$m and $L_2 = (0.5, 12.5)$m.

- The number of sensors $N_d = 20$, while the UCA radius $r = \frac{N_d \frac{\lambda}{2}}{2\pi}$, and $P = 100$ snapshots are collected unless specified otherwise.

- The area of interest is set as $[-20, 20]$m along both x-axis and y-axis. In the on-grid method and initial step of the off-grid method, 2m is used as the stepsize for the grid unless specified otherwise. For grid refinement, a new grid with stepsize 0.2m is formed around a distance of 1m to either side of the estimated location from the initial step.

4. Target Localization Based on Distributed Array Networks – Simulation Results

- The signal to noise ratio (SNR) is 20 dB.

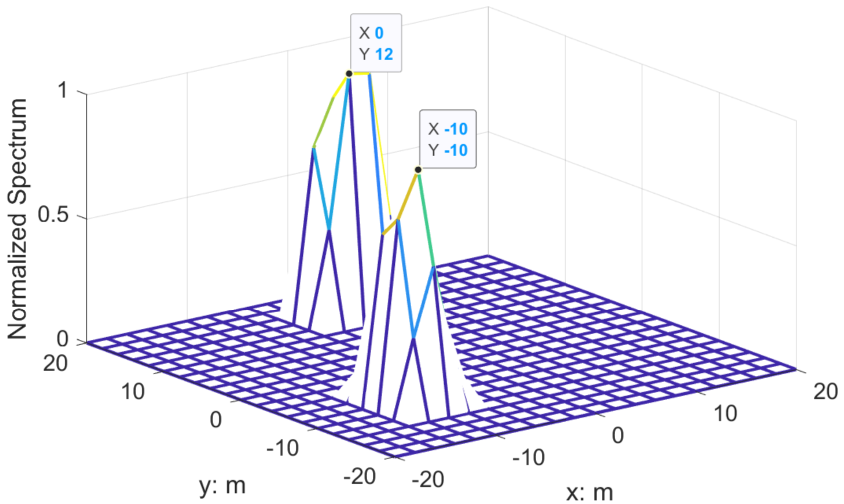


Fig. 5. Result by the on-grid method.

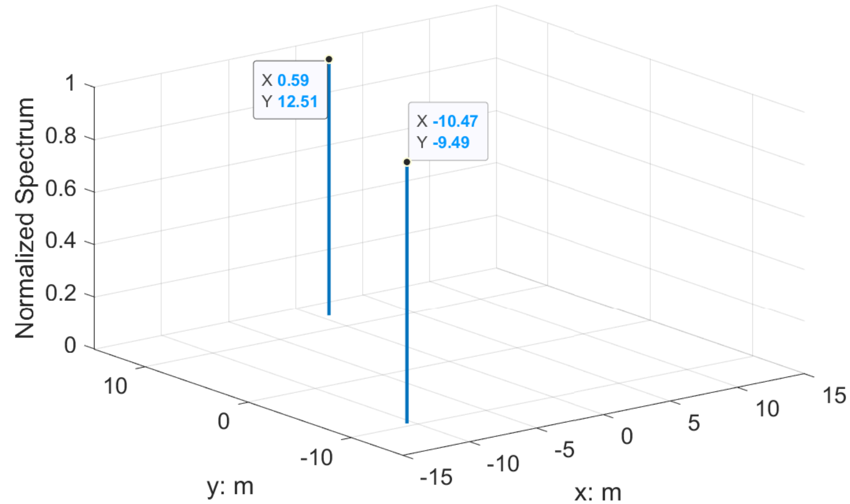
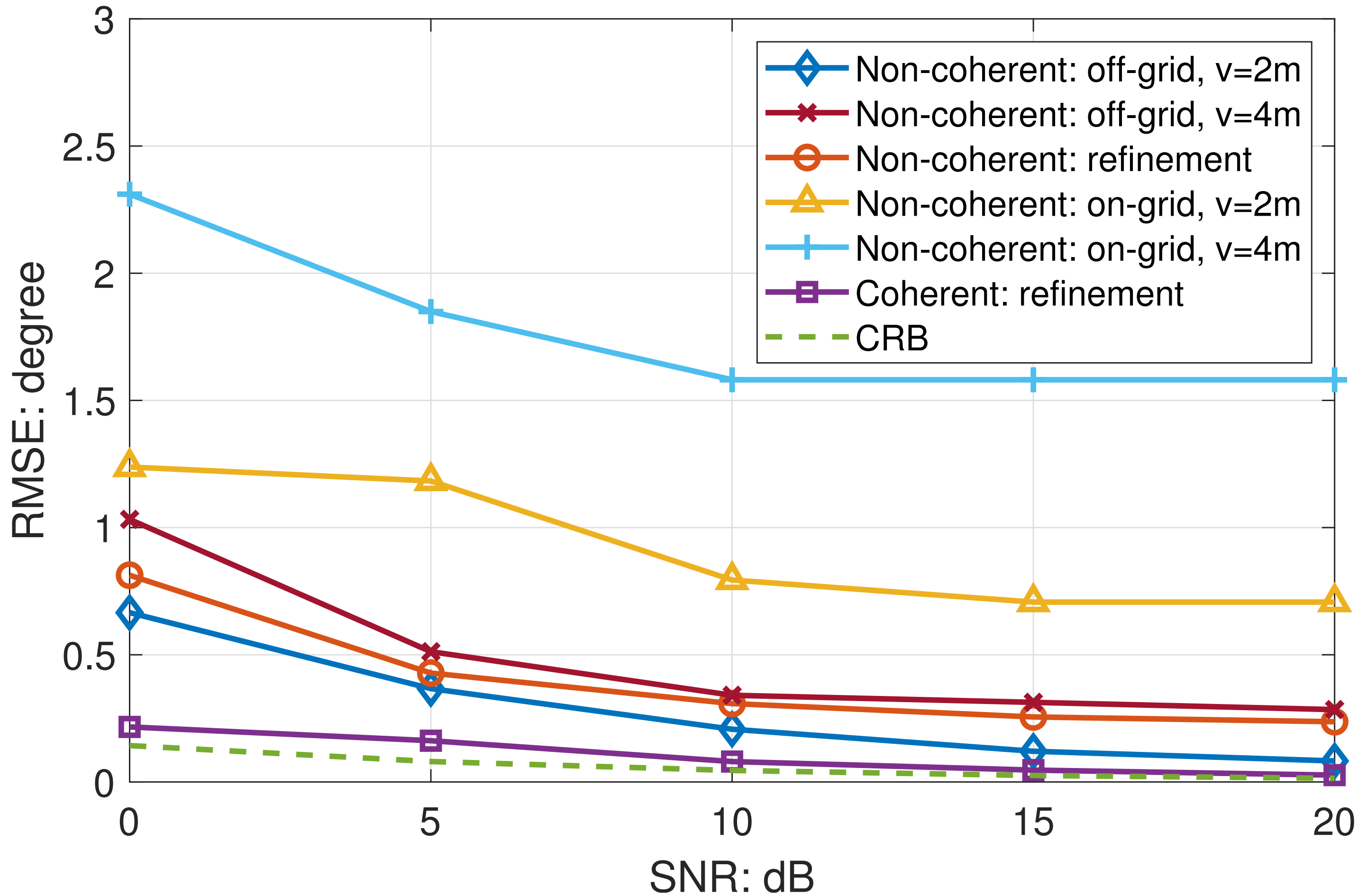


Fig. 6. Result by the off-grid method.

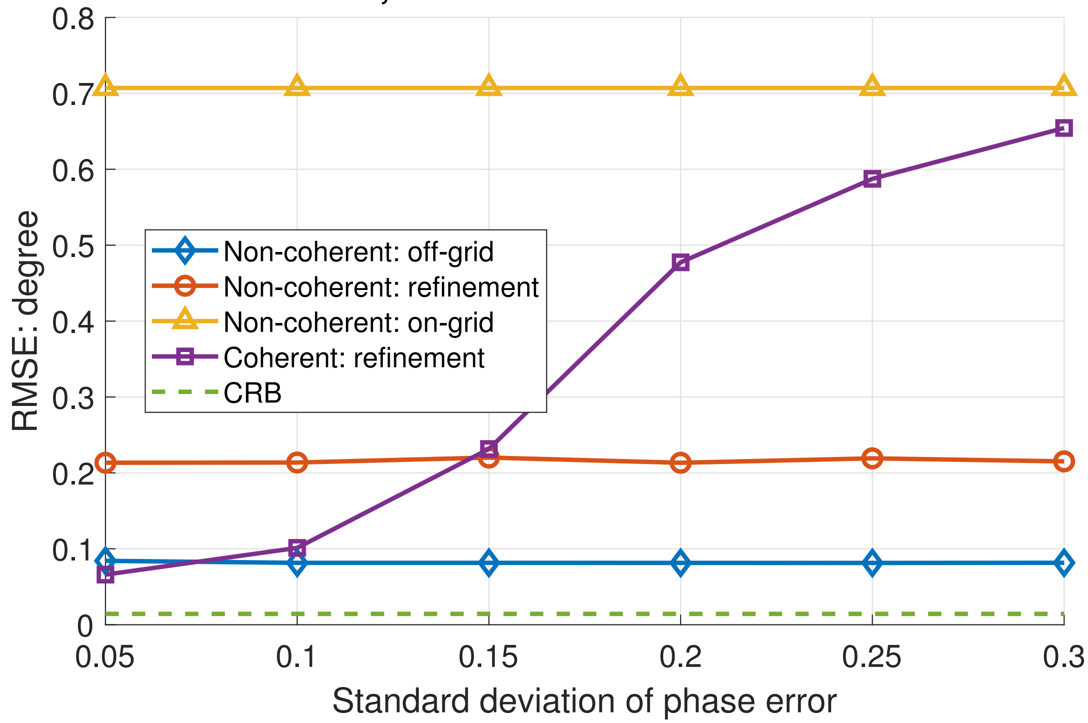
4. Target Localization Based on Distributed Array Networks – Simulation Results

- RMSE versus SNR over 100 trials without phase errors.



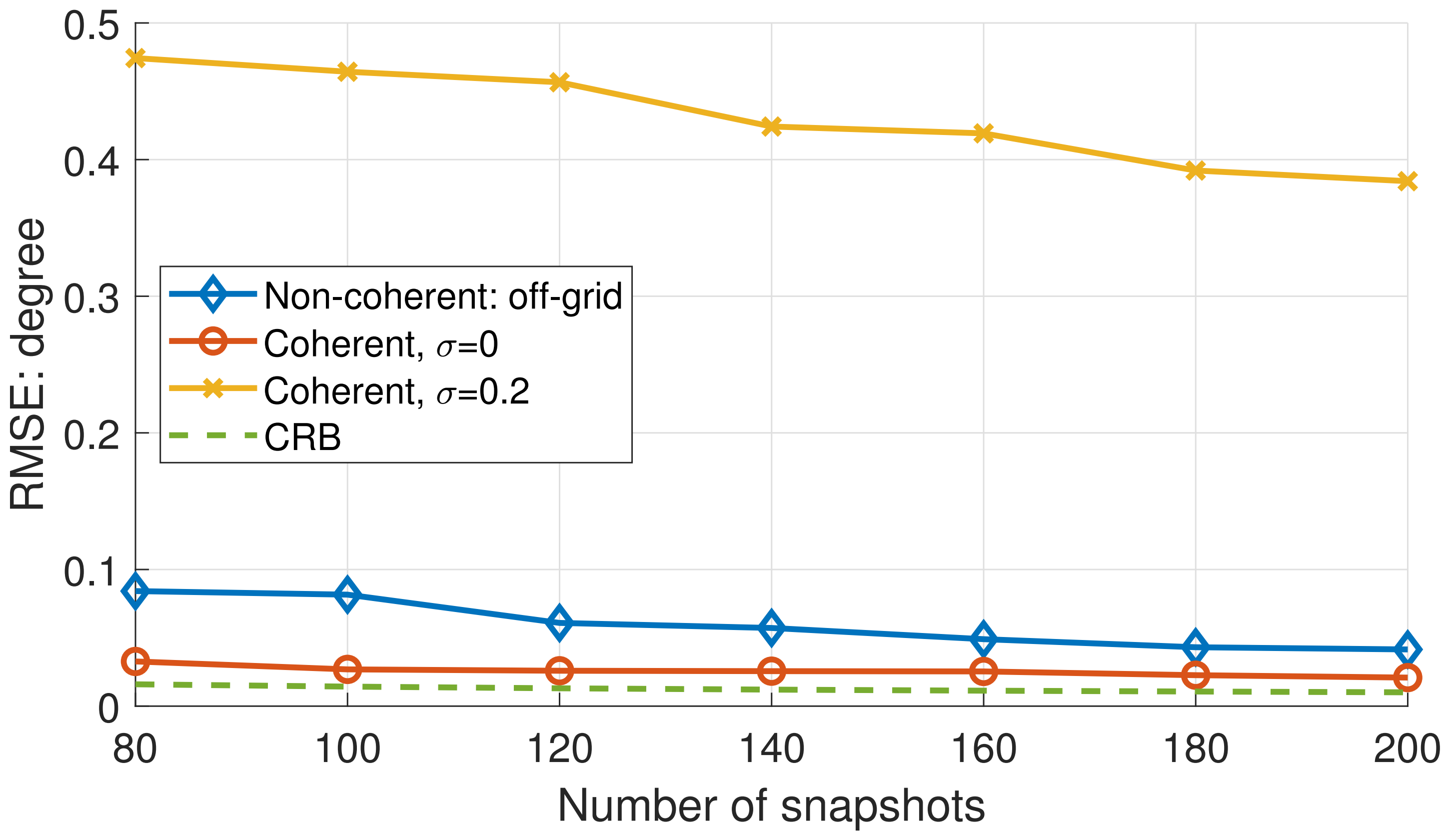
4. Target Localization Based on Distributed Array Networks – Simulation Results

- Performance in the presence of sensor phase errors, which are modeled as $\mathbf{Z}_d = |\mathbf{E}_d\mathbf{A}_d\mathbf{S}_d| + \mathbf{N}_d$, where $\mathbf{E}_d$ is an $N_d \times N_d$ diagonal matrix with each entry being a unit complex variable with a random phase term generated independently from the Gaussian distribution with standard derivation σ; SNR=20dB.



4. Target Localization Based on Distributed Array Networks – Simulation Results

- RMSEs versus number of snapshots with phase error $\sigma = 0$ and $\sigma = 0.2$; SNR=20dB.



5. Conclusions

- In this talk, a new class of solutions have been presented for effective target direction finding and localization without relying on the phase information of the received array signals, leading to robust solutions to the problem based on magnitude-only measurements.

- The non-coherent direction finding problem was first studied and inherent ambiguities for DOA estimation associated with phaseless measurements were discussed in detail with the ULA as a representative example; two structures were provided to tackle the ambiguities issue, including a dual-array structure and the UCA.

- The non-coherent DOA estimation problem can be formulated as a group sparse phase retrieval problem and solved by the proximal gradient method after transforming the original non-convex cost function into its convex surrogate via the PRIME technique, leading to an algorithm called ToyBar.

5. Conclusions

- Then the source/target localization problem was studied based on distributed sensor array networks with phaseless measurements.

- The resultant non-coherent source localization problem was formulated into a joint sparse phase retrieval form and can also be solved using existing group-sparsity based algorithms such as ToyBar.

- As target locations are obtained jointly and directly from the received signals, the proposed solution is also robust against individual measurement errors.

- It is also possible to develop off-grid solutions for the proposed methods and for detail, please refer to our recent publication [14].

- Further extension to the wideband case can be achieved through the convolutional sparse coding framework [15, 16].

References

[1] Y. Shechtman, Y. C. Eldar, O. Cohen, H. N. Chapman, J. w. Miao, and M. Segev, “Phase retrieval with application to optical imaging: A contemporary overview,” *IEEE Signal Processing Magazine*, vol. 32, no. 3, pp. 87–109, 2015.

[2] H. Kim, A. M. Haimovich, and Y. C. Eldar, “Non-coherent direction of arrival estimation from magnitude only measurements,” *IEEE Signal Processing Letters*, vol. 22, no. 7, pp. 925–929, July 2015.

[3] Z. Wan and W. Liu, “Non-coherent DOA estimation based on a dual-array structure without reference signals,” in *Prof. IEEE International Workshop on Computational Advances in Multi-Sensor Adaptive Processing*, 2019, pp. 46–50.

[4] Z. Y. Wan and W. Liu, “Non-coherent DOA estimation via proximal gradient based on a dual-array structure,” *IEEE Access*, vol. 9, pp. 26 792–26 801, Feb. 2021.

[5] C. R. Karanam, B. Korany, and Y. Mostofi, “Magnitude-based angle of arrival estimation, localization and target tracking,” in *Proc. of the*

ACM/IEEE International Conference on Information Processing in Sensor Networks, Porto, April 2018, pp. 254–265.

[6] Z. Wan and W. Liu, "Non-coherent DOA estimation of off-grid signals with uniform circular arrays," in *Proc. IEEE International Conference on Acoustics, Speech, and Signal Processing*, Toronto, Canada, Apr. 2021, pp. 4370–4374.

[7] T. Qiu, P. Babu, and D. P. Palomar, "PRIME: Phase retrieval via majorization-minimization," *IEEE Transactions on Signal Processing*, vol. 64, no. 19, pp. 5174–5186, June 2016.

[8] A. Beck and M. Teboulle, "Gradient-based algorithms with applications to signal-recovery problems," in *Convex Optimization in Signal Processing and Communications*, D. P. Palomar and Y. C. Eldar, Eds. Cambridge University Press, 2009, pp. 42–88.

[9] N. Parikh and S. Boyd, "Proximal algorithms," *Foundations and Trends in Optimization*, vol. 1, no. 3, pp. 127–239, Jan 2014.

[10] W. Deng, W. Yin, and Y. Zhang, "Group sparse optimization by alternating direction method," in *Proc. of SPIE 8858, Wavelets and Sparsity XV, 88580R*, San Diego, US, 2013.

[11] Y. Nesterov, "A method for solving the convex programming

problem with convergence rate O$(1/k^2)$," *Proc. of the USSR Academy of Sciences*, vol. 269, pp. 543–547, 1983.

[12] Q. Shen, W. Liu, L. Wang, and Y. Liu, "Group sparsity based localization for far-field and near-field sources based on distributed sensor array networks," *IEEE Transactions on Signal Processing*, vol. 68, pp. 6493–6508, 2020.

[13] Z. Wan, W. Liu, and P. Willett, "Non-coherent source localization with distributed sensor array networks," in *Proc. IEEE Sensor Array and Multichannel Signal Processing Workshop (SAM)*, 2022, pp. 86–90.

[14] ——, "Target localization based on distributed array networks with magnitude-only measurements," *IEEE Transactions on Aerospace and Electronic Systems*, vol. 59, no. 6, pp. 9704–9710, 2023.

[15] Z. Wan and W. Liu, "Time-domain wideband DOA estimation under the convolutional sparse coding framework," *IEEE Signal Processing Letters*, vol. 29, pp. 274–278, 2022.

[16] ——, "Wideband DOA estimation with magnitude-only measurements," in *Proc. International Symposium on Circuits and Systems (ISCAS)*, 2023.