Target Detection in Heterogeneous Clutter

Elias Aboutanios

Introduction

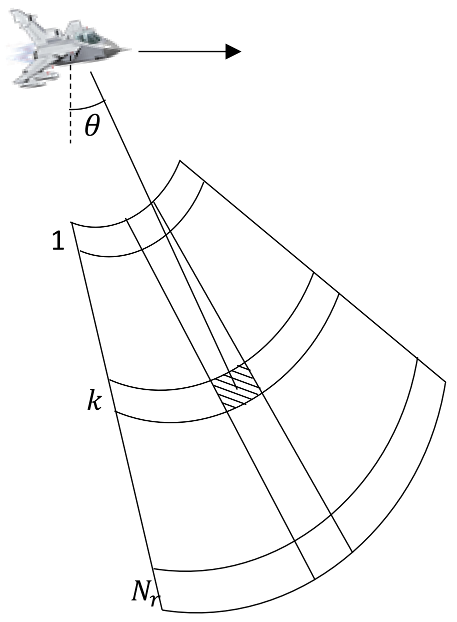
- Effective target detection requires the suppression of clutter and interference
- Achieved by applying adaptive filter to the range cell under test (CUT)
- This requires knowledge of clutter and interference covariance matrix
  - Covariance matrix estimated using training data
  - Assume training data is statistically homogeneous with CUT
- Homogeneity assumption violated in practice → loss in detection performance
  - Discretes in training data
  - Mismatch in clutter power
  - Non-Gaussian clutter
  - Clutter Motion
- Need detection approaches that are robust to clutter heterogeneity

Outline

- Airborne Radar Signal Model
- Target Detection and the Two-Data Set Detectors
- Clutter Heterogeneity
- Detection Approaches for Heterogeneous Clutter
- Single Data Set Algorithms and hybrid Detectors
- Texture Estimation and Normalisation Detector
- The Multistage Wiener Filter
- Rank Estimation Approaches for the MWF

Airborne Radar Signal Model

- Airborne radar travelling at velocity v_p
- N antennas with inter-element spacing d
- Coherent Pulse Interval (CPI)
  - ➢comprising M pulses
  - ➢Pulse repetition frequency f_{PRF}
- Radar collects samples from N_r range gates
- Focus on range k. Clutter patch at angle θ
  - Spatial frequency $f_s(\theta) = \frac{d}{\lambda} \sin\theta$
  - Doppler frequency $f_d(\theta) = \frac{v_p}{\lambda} \sin\theta$



Airborne Radar Signal Model

- Steering matrix of clutter patch is

$$\mathbf{S}_{\mathrm{c}}(\theta) = \mathbf{s}_{\mathrm{s}}(\theta)\mathbf{s}_{\mathrm{t}}^{T}(\theta)$$

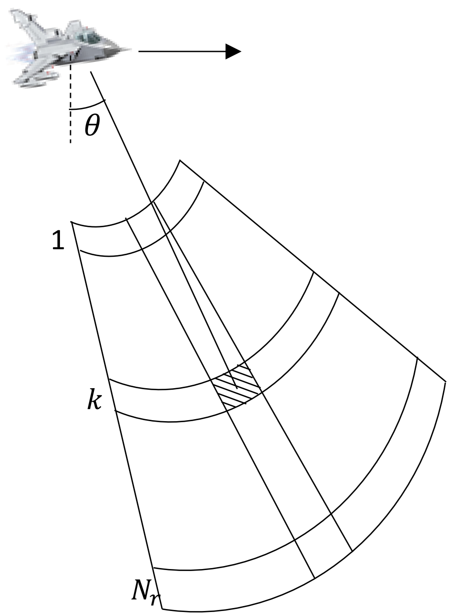
where

  - $\mathbf{s}_{\mathrm{s}}(\theta) = [1 \quad e^{j2\pi f_s} \ \ldots \quad e^{j2\pi(N-1)f_s}]^T$ and
  - $\mathbf{s}_{\mathrm{t}}(\theta) = [1 \quad e^{j2\pi f_d T_r} \ \ldots \quad e^{j2\pi(M-1)f_d T_r}]^T$

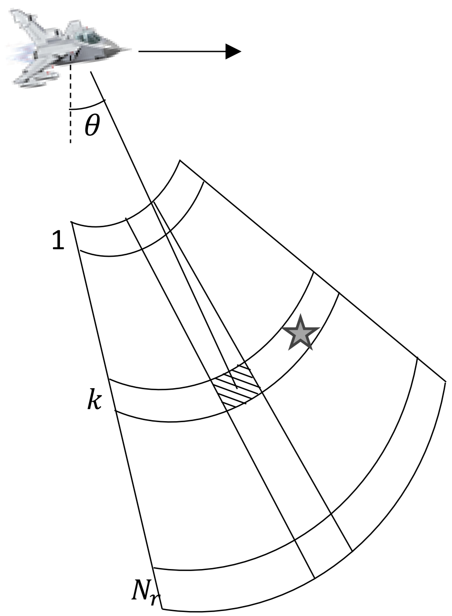
- Complex reflectivity $\rho(\theta)$
- Then clutter return for range gate k is

$$\mathbf{C}_k = \int_{-\pi}^{\pi} \rho(k,\theta) g(\theta) \mathbf{S}_{\mathrm{c}}(\theta) d\theta$$

where $g(\theta)$ is the two-way beampattern



Airborne Radar Signal Model



- Target present in cell k with reflectivity α
- Target steering vector $\mathbf{s} = \mathbf{s}_\mathrm{s}\mathbf{s}_\mathrm{t}^T$ where
  - $\mathbf{s}_{\mathrm{s,T}} = [1 \quad e^{j2\pi\nu_\mathrm{T}} \; \dots \quad e^{j2\pi(N-1)\nu_\mathrm{T}}\,]^T$ and
  - $\mathbf{s}_{\mathrm{t,T}} = [1 \quad e^{j2\pi f_{d,\mathrm{T}}T_r} \; \dots \quad e^{j2\pi(M-1)f_{d,\mathrm{T}}T_r}\,]^T$
- The radar received signal model for range k becomes

$$\mathbf{X}_k = \alpha\mathbf{S} + \mathbf{C}_k + \mathbf{N}_k$$

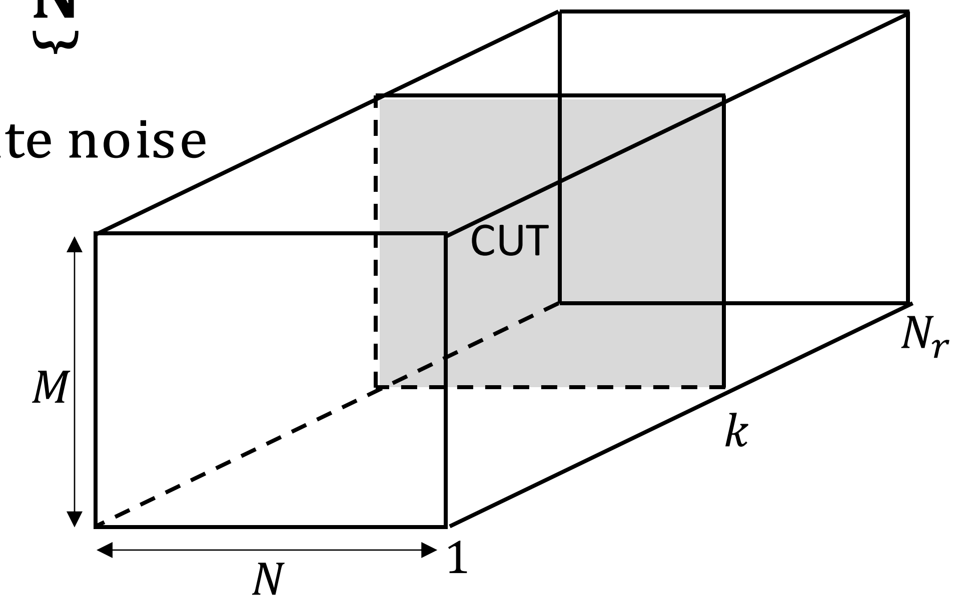
where $\mathbf{N}_k$ is a matrix of white Gaussian noise

- Stacking the matrices $\mathbf{X}_k$, $k = 1, \dots, K$ to give the well-known data cube

Target Detection

- The received signal for range under test (CUT)

$$\mathbf{X} = \underbrace{\alpha\mathbf{S}}_{\text{target}} + \underbrace{\mathbf{C}}_{\text{clutter}} + \underbrace{\mathbf{N}}_{\text{white noise}}$$



- Vectorise to give length-L $(= MN)$ received vector

$$\mathbf{x} = \mathrm{vec}(\mathbf{X})$$
$$= \alpha\mathbf{s} + \mathbf{c} + \mathbf{n}$$

- Assume reflectivities $\rho \sim \mathcal{CN}(0,1)$
- Clutter plus noise, $\boldsymbol{\varsigma} = \mathbf{c} + \mathbf{n}$, Gaussian distributed

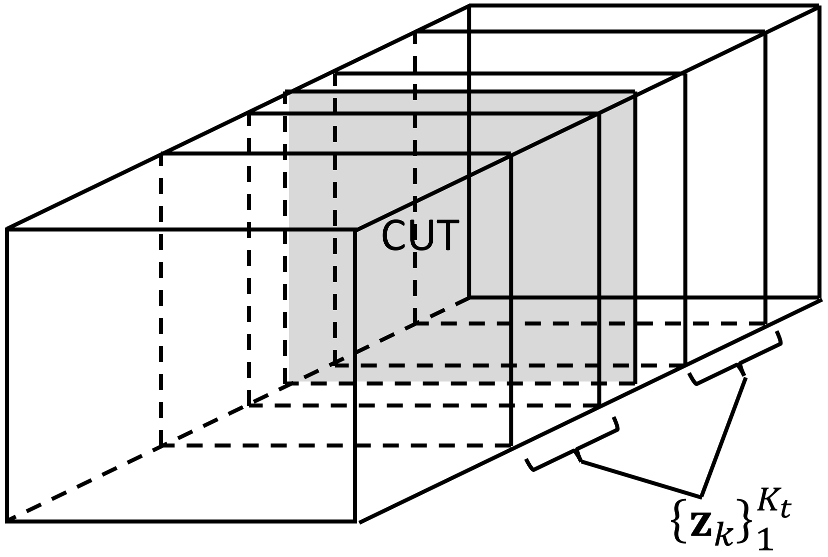
$$\boldsymbol{\varsigma} \sim \mathcal{CN}_L(\mathbf{0}, \mathbf{R})$$

- Formulate detection problem as hypothesis test for presence of target

Target Detection

- Hypothesis test for $|\alpha| > 0$ vs $|\alpha| = 0$

$$H_0: \ \mathbf{x} = \boldsymbol{\varsigma}$$
$$H_1: \ \mathbf{x} = \alpha\mathbf{s} + \boldsymbol{\varsigma}$$



- Obtain suitable statistic and compare to a threshold

$$y \underset{H_0}{\overset{H_1}{\gtrless}} \gamma$$

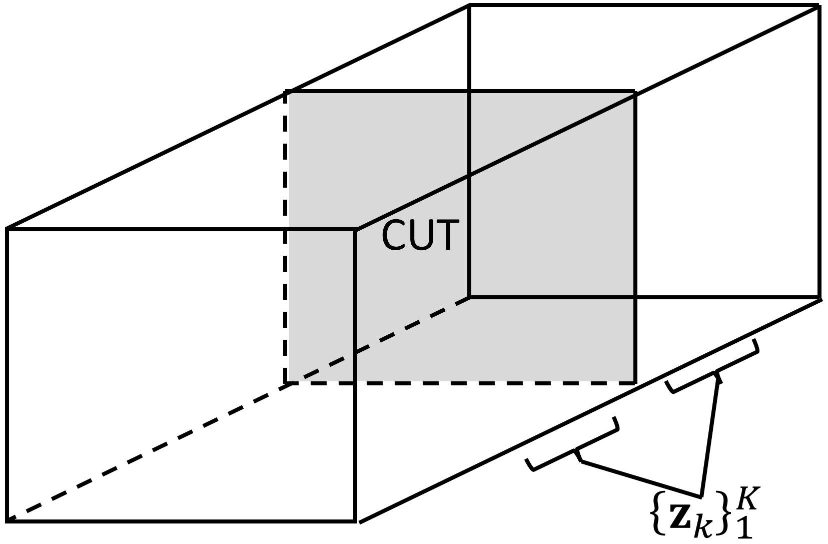
- Likelihood ratio test maximises probability of detection for fixed probability of false alarm
- Then

$$y = \frac{f_x(\mathbf{x}|H_1)}{f_x(\mathbf{x}|H_0)} \quad \text{equivalently} \quad y = \ln f(\mathbf{x}|H_1) - \ln f(\mathbf{x}|H_0)$$

Detection Schemes - Two Data Set Algorithms

- Optimal detector is matched filter

$$y = \frac{|\mathbf{s}^H \mathbf{R}^{-1} \mathbf{x}|^2}{\mathbf{s}^H \mathbf{R}^{-1} \mathbf{s}}$$



- Requires knowledge of true covariance matrix $\mathbf{R}$
- Target-free homogeneous training data set $\mathbf{Z}$ of size K
- Sample covariance matrix $\widehat{\mathbf{R}} = \frac{1}{K} \sum_{k=1}^{K} \mathbf{z}_k \mathbf{z}_k^{\mathrm{H}}$
- Practical two-data set (TDS) detectors

$$y_{\mathrm{GLRT}} = \frac{|\mathbf{s}^H \widehat{\mathbf{R}}^{-1} \mathbf{x}|^2}{(\mathbf{s}^H \widehat{\mathbf{R}}^{-1} \mathbf{s})(1 + \frac{1}{K} \mathbf{x}^H \widehat{\mathbf{R}}^{-1} \mathbf{x})}$$

Generalised Likelihood Ratio Test

$$y_{\mathrm{AMF}} = \frac{|\mathbf{s}^H \widehat{\mathbf{R}}^{-1} \mathbf{x}|^2}{\mathbf{s}^H \widehat{\mathbf{R}}^{-1} \mathbf{s}}$$

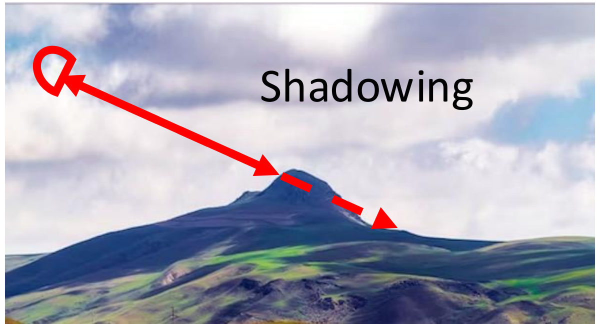
Adaptive Matched Filter

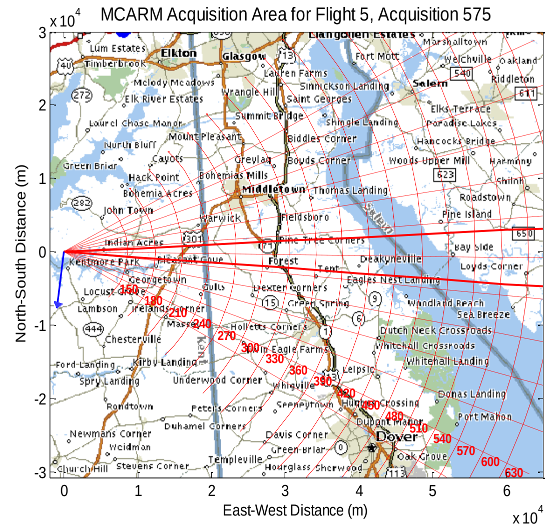
$$y_{\mathrm{NAMF}} = \frac{|\mathbf{s}^H \widehat{\mathbf{R}}^{-1} \mathbf{x}|^2}{(\mathbf{s}^H \widehat{\mathbf{R}}^{-1} \mathbf{s})(\mathbf{x}^H \widehat{\mathbf{R}}^{-1} \mathbf{x})}$$

Normalised AMF

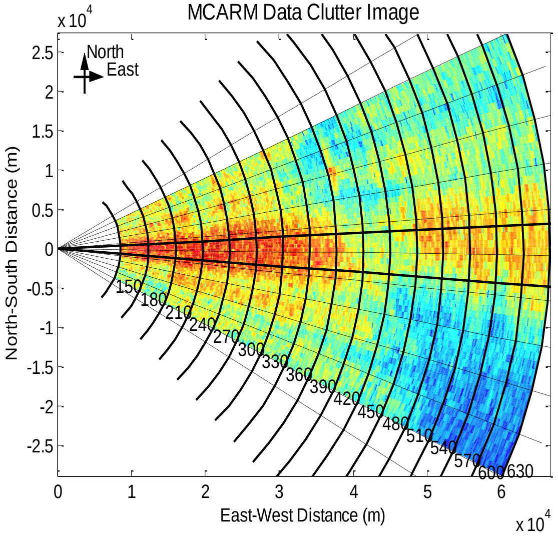
Clutter Heterogeneity

- Assumption on $\mathbf{Z}$
  - ➢ Snapshots, $\mathbf{z}_k$, are free from targets
  - ➢ independent and identically distributed (iid)
  - ➢ have same distribution as $\mathbf{x}$





- Any, or all, assumptions violated in practice
- Heterogeneity arises due to
  - ➢ Discretes in training data
  - ➢ Mismatch in clutter power
  - ➢ Non-Gaussian clutter
  - ➢ Clutter Motion (non-stationary)



Detection in Heterogeneous Clutter

- Derive detectors for particular distributions, e.g.
  - K-distributed clutter
  - Compound K + Gaussian clutter
- Let $\mathbf{R} = \tau\mathbf{G}$ where $\mathbf{G}$ fixed and τ is a random variable
- K-distributed clutter

$\rightarrow \tau$ follows a Gamma distribution with mean μ and shape parameter ν

- Optimum detector

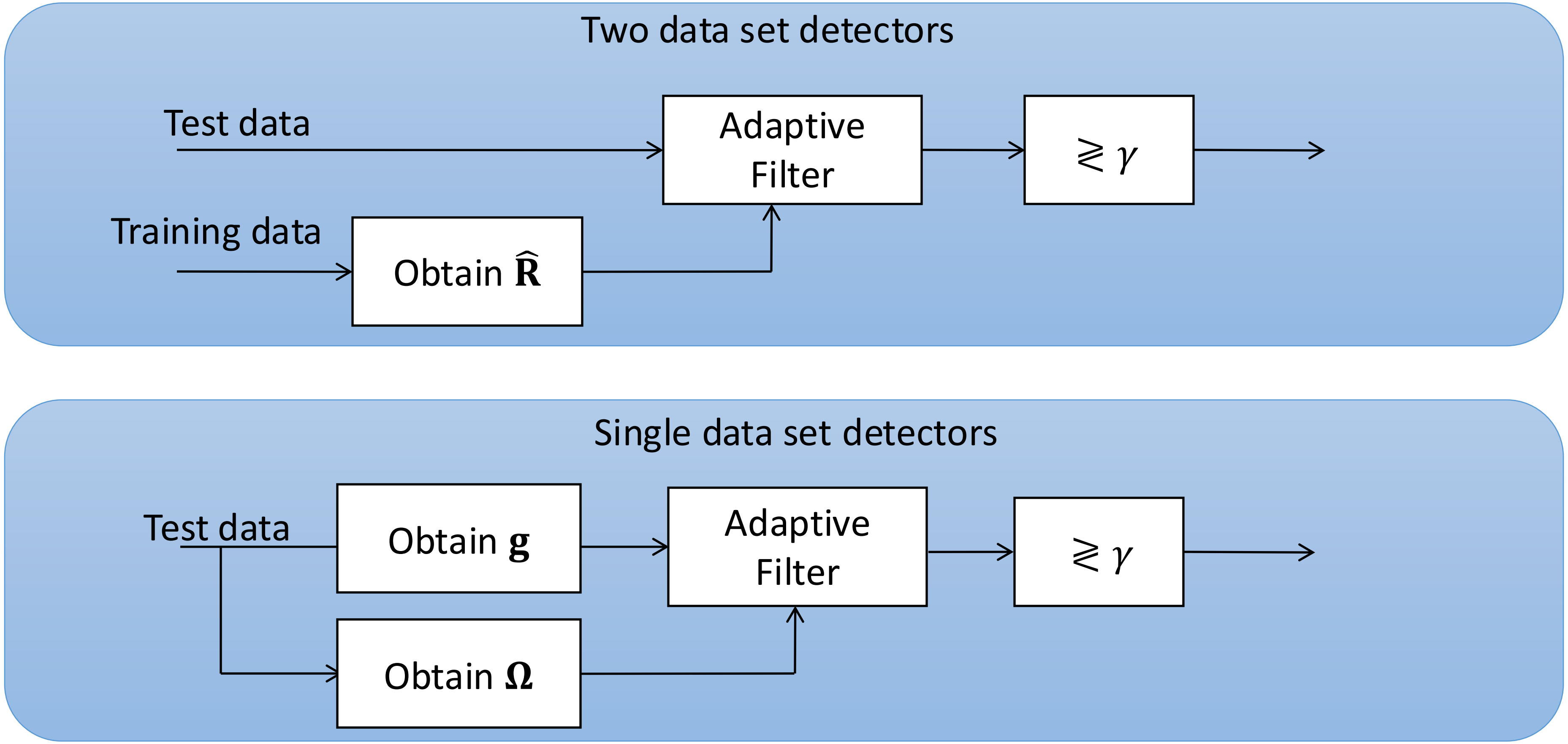
$$y_K = \frac{\left(\mathbf{x}^H\mathbf{G}^{-1}\mathbf{x}\right)(\mathbf{s}^H\mathbf{G}^{-1}\mathbf{s}) - |\mathbf{s}^H\mathbf{G}^{-1}\mathbf{x}|^2}{\mathbf{s}^H\mathbf{G}^{-1}\mathbf{s}}$$

- Requires knowledge of $\mathbf{G}$

Detection in Heterogeneous Clutter

- Knowledge-aided detection
  - ➢ Use knowledge of platform and radar parameters
  - ➢ Use digital terrain and elevation data and land cover and use
- Training data pre-screening
  - ➢ non-homogeneity detection (NHD) strategies e.g. Generalised Inner Product (GIP)
- Reduce training data (sample support) requirements:
  - ➢ Rank reduced detectors
  - ➢ Sparsity techniques
- Remove training data requirement:
  - Deterministic Direct Domain
  - Single Data Set detectors

Single Data Set Detectors



Single Data Set Detectors

- Single Data Set (SDS) uses data only from cell under test (CUT)
- Sliding window of size $P \times Q$ to partition CUT giving matrix $\mathbf{X}_\mathrm{T}$ of size $PQ \times K_T$

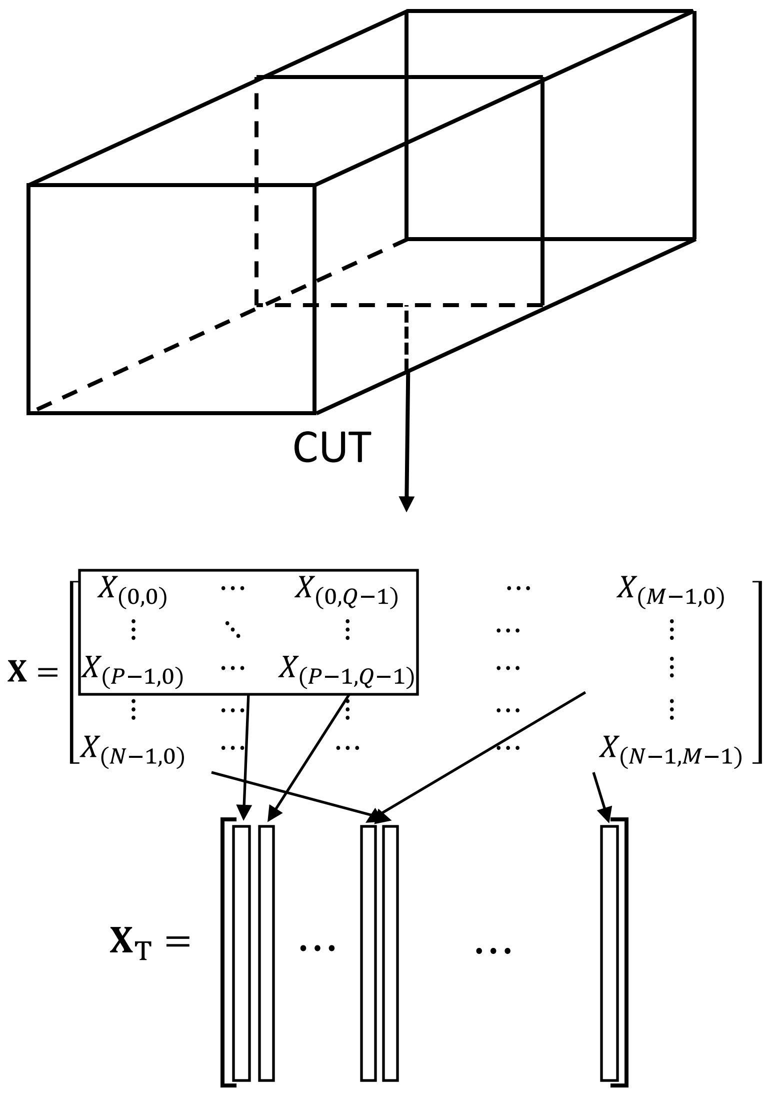
$$K_T = (N - P + 1)(M - Q + 1)$$

- Mean vector and covariance estimate

$$\mathbf{g} = \frac{1}{K_\mathrm{T}} \mathbf{X}_\mathrm{T} \mathbf{t}^* \text{ and } \mathbf{\Omega} = \frac{1}{K_\mathrm{T} - 1} \left(\mathbf{X}_\mathrm{T} \mathbf{X}_\mathrm{T}^\mathrm{H} - \mathbf{g}\mathbf{g}^\mathrm{H} \right)$$

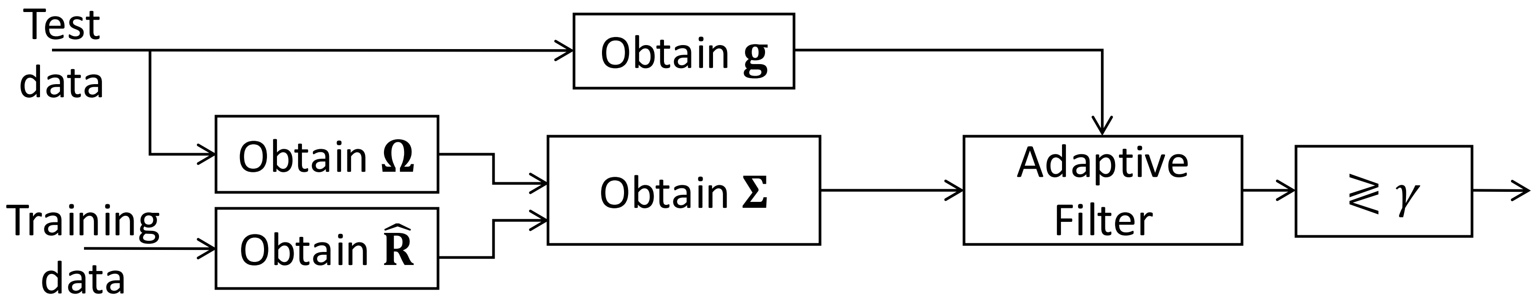
- The SDS statistic

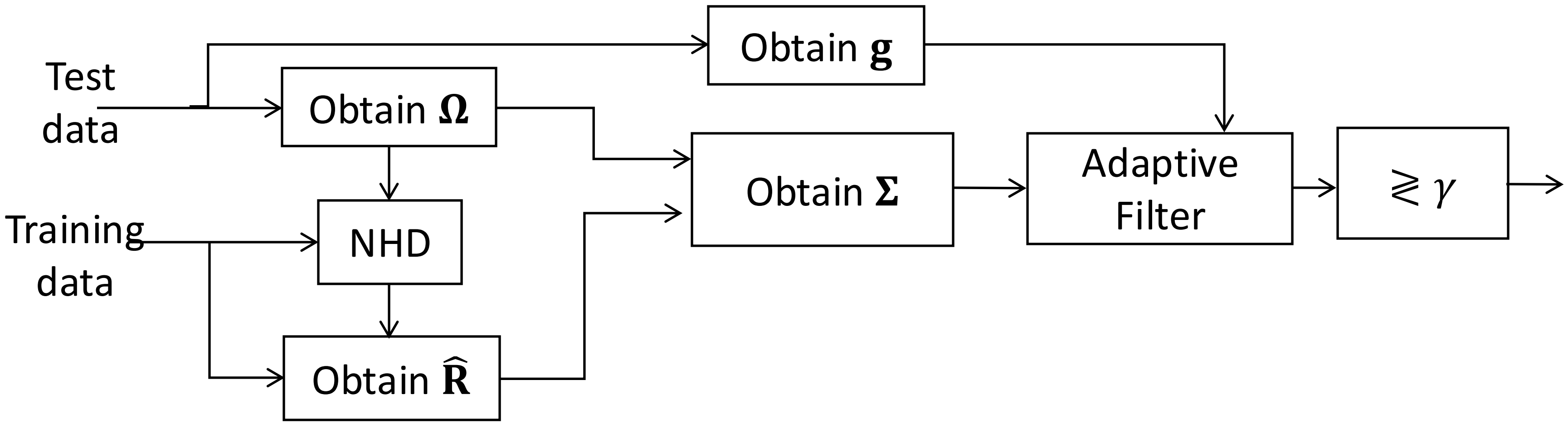
$$y_\mathrm{SDS} = \frac{\left| \mathbf{s}^H \, \widehat{\mathbf{\Omega}}^{-1} \mathbf{g} \right|^2}{\mathbf{s}^H \, \widehat{\mathbf{\Omega}}^{-1} \mathbf{s}}$$



Hybrid Detection Approaches

- Training data usually not completely heterogeneous
- Combine the TDS and SDS approaches → take advantage of both





Fixed Scale Hybrid

- Assume training data is statistically homogeneous
- Combine estimates $\mathbf{\Omega}$ and $\widehat{\mathbf{R}}$
- Fixed Scale Hybrid: covariance matrix estimate and detection statistic

$$\widehat{\mathbf{\Sigma}}_{\mathrm{FSH}} = \frac{(K_T - 1)\mathbf{\Omega} + K_t\widehat{\mathbf{R}}}{K_T + K_t - 1}, \text{and } y_{\mathrm{FSH}} = \frac{\left|\mathbf{s}^H\widehat{\mathbf{\Sigma}}_{\mathrm{FS}}^{-1}\mathbf{g}\right|^{\mathbf{2}}}{\mathbf{s}^H\widehat{\mathbf{\Sigma}}_{\mathrm{FS}}^{-1}\mathbf{s}}$$

- Fixed scale hybrid gives improved performance in homogeneous environments
- Degrades in heterogeneous environments due to inclusion of training data
- Therefore, screen data for heterogeneity first

Variable Scale Hybrid

- Variable scale hybrid detector
- Use $b_k, k = 1, \ldots, K_t$ as a measure of heterogeneity of range k wrt CUT
- Covariance matrix estimate $\widehat{\mathbf{R}}$ becomes

$$\widehat{\mathbf{R}} = \frac{1}{\mathrm{tr}(\mathbf{B})}\mathbf{Z}\mathbf{B}\mathbf{Z}^H \text{ where } \mathbf{B} = diag(\mathbf{b})$$

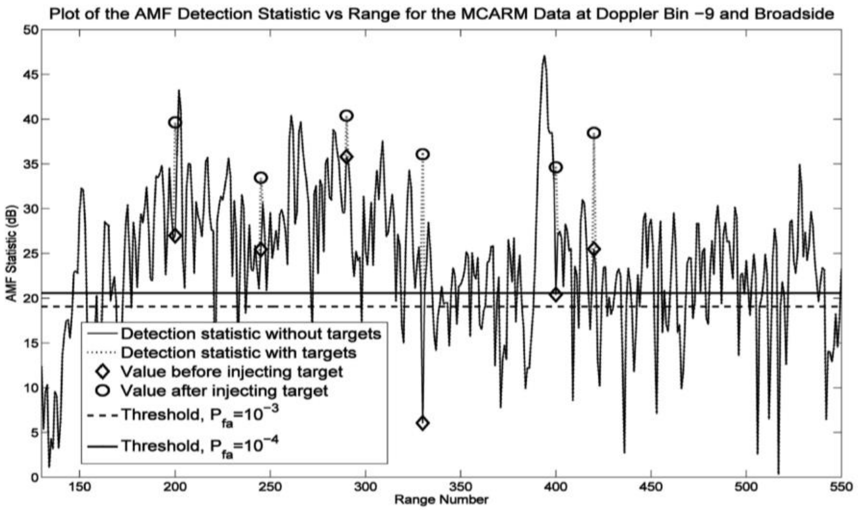
- Variable Scale Hybrid covariance matrix estimate and detection statistic

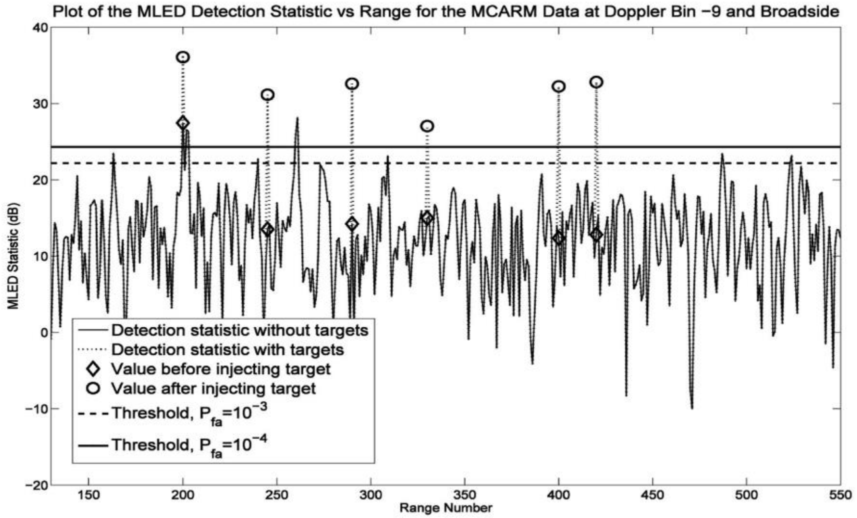
$$\widehat{\boldsymbol{\Sigma}}_{\mathrm{VSH}} = \frac{(K_T - 1)\boldsymbol{\Omega} + \mathrm{tr}(\mathbf{B})\widehat{\mathbf{R}}}{K_T + \mathrm{tr}(\mathbf{B}) - 1}, \text{and } y_{VSH} = \frac{\left|\mathbf{s}^H \widehat{\boldsymbol{\Sigma}}_{\mathrm{VSH}}^{-1} \mathbf{g}\right|^{\mathbf{2}}}{\mathbf{s}^H \widehat{\boldsymbol{\Sigma}}_{\mathrm{VSH}}^{-1} \mathbf{s}}$$

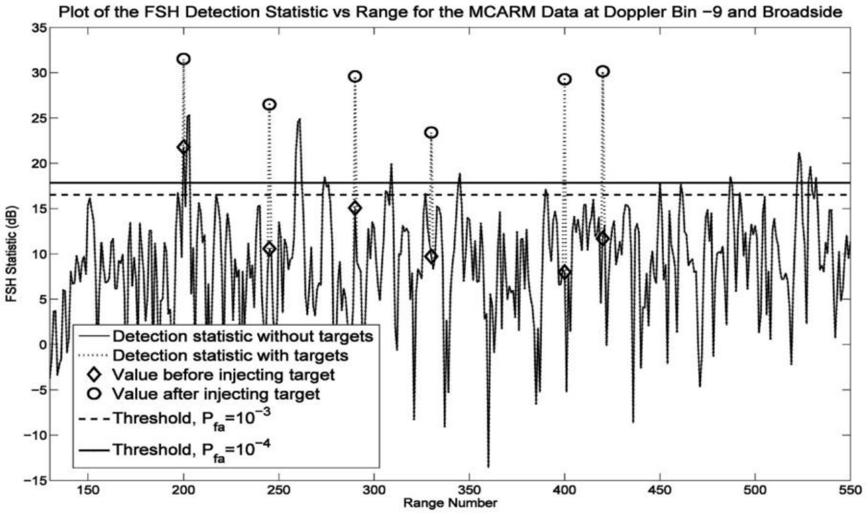
- Using Generalised Inner Product for screening the data we have

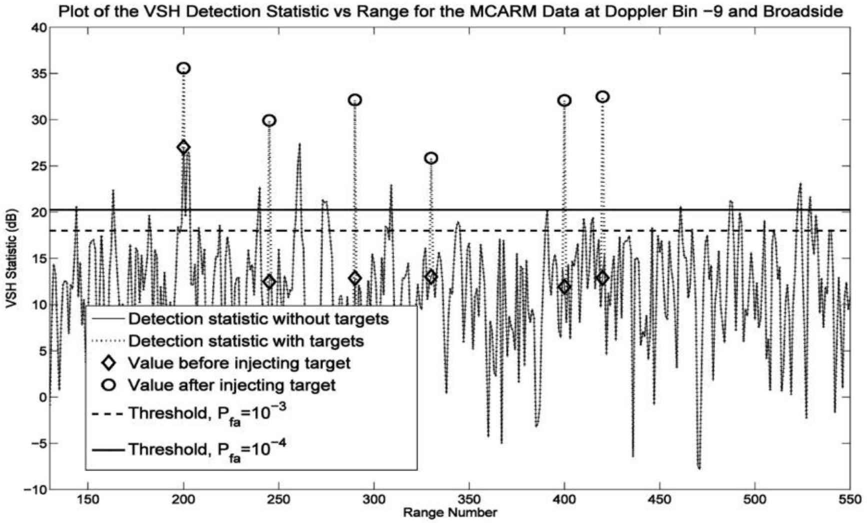
$$b_k = \begin{cases} 1, & if\ \nu_L \le p_k \le \nu_U \\ 0, & \text{otherwise} \end{cases} \quad \text{where } p_k = \mathbf{z}_k^H \mathbf{Q}^{-1} \mathbf{z}$$

Performance Evaluation (MCARM Data)









E. Aboutanios and B. Mulgrew, "Hybrid Detection Approach for STAP in Heterogeneous Clutter," in *IEEE Transactions on Aerospace and Electronic Systems*, vol. 46, no. 3, pp. 1021-1033, July 2010, doi: 10.1109/TAES.2010.5545171.

Texture Estimation and Normalisation Detector

- Maritime clutter modelled using a speckle matrix and a texture
- K-distributed clutter

$$\mathbf{x} \sim CN(\mathbf{0}, \tau_T \mathbf{G})$$

- For each training snapshot we have

$$\mathbf{z}_k \sim CN(\mathbf{0}, \tau_k \mathbf{G})$$

- Partition each snapshot to give $P \times Q$ matrix $\mathbf{Z}_k$ and obtain $\widehat{\mathbf{R}}_k$
- Now

$$\begin{aligned} \boldsymbol{Q}^{-1}\widehat{\boldsymbol{R}}_k &\approx \frac{\tau_k}{\tau_T}\widehat{\mathbf{G}}^{-1}\widehat{\mathbf{G}}_k \\ &\approx \frac{\tau_k}{\tau_T}\mathbf{I} \end{aligned}$$

- Put $\zeta_k = \frac{1}{M}\operatorname{tr}\left(\mathbf{Q}^{-1}\widehat{\mathbf{R}}_k\right)$, then ζ_k is an estimate of $\frac{\tau_k}{\tau_T}$

Texture Estimation and Normalisation Detector

- Use ζ_k to normalise the texture in the training snapshots
- Define rescaled covariance matrix

$$\widetilde{\mathbf{R}}_k = \frac{1}{\zeta_k}\widehat{\mathbf{R}}_k \approx \tau_T \widehat{\mathbf{G}}_k$$

- This normalises the heterogeneity resulting from texture variations
- Finally, covariance matrix estimate and detection statistic become

$$\widehat{\mathbf{\Sigma}}_{\mathrm{TEN}} = \frac{1}{K + K_T - 1}\left\{(K_T - 1)\widehat{\mathbf{Q}} + \sum_k \widetilde{\mathbf{R}}_k\right\}$$

- Detection statistic is

$$y_{\mathrm{TEN}} = \frac{\left|\mathbf{s}^H \widehat{\mathbf{\Sigma}}_{\mathrm{TEN}}^{-1} \mathbf{g}\right|^2}{\mathbf{s}^H \widehat{\mathbf{\Sigma}}_{\mathrm{TEN}}^{-1} \mathbf{s}}$$

UNSW SYDNEY

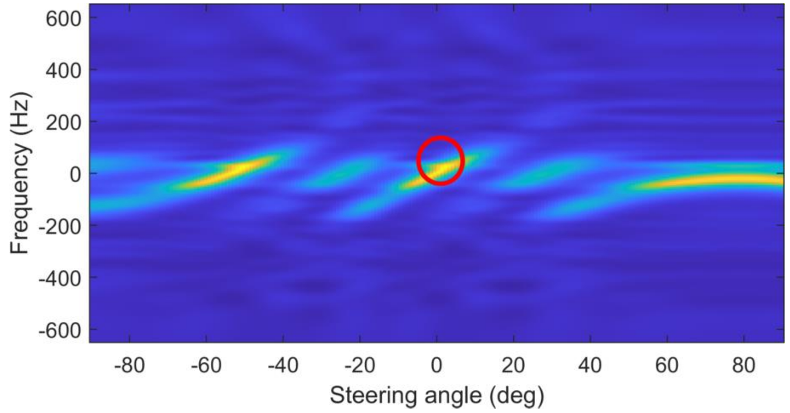
Performance Evaluation – Simulated Sea Clutter

- Simulated data set uses L-band radar and K-distributed clutter
- Shape parameter a, and scale parameter b, related to mean texture value μ by

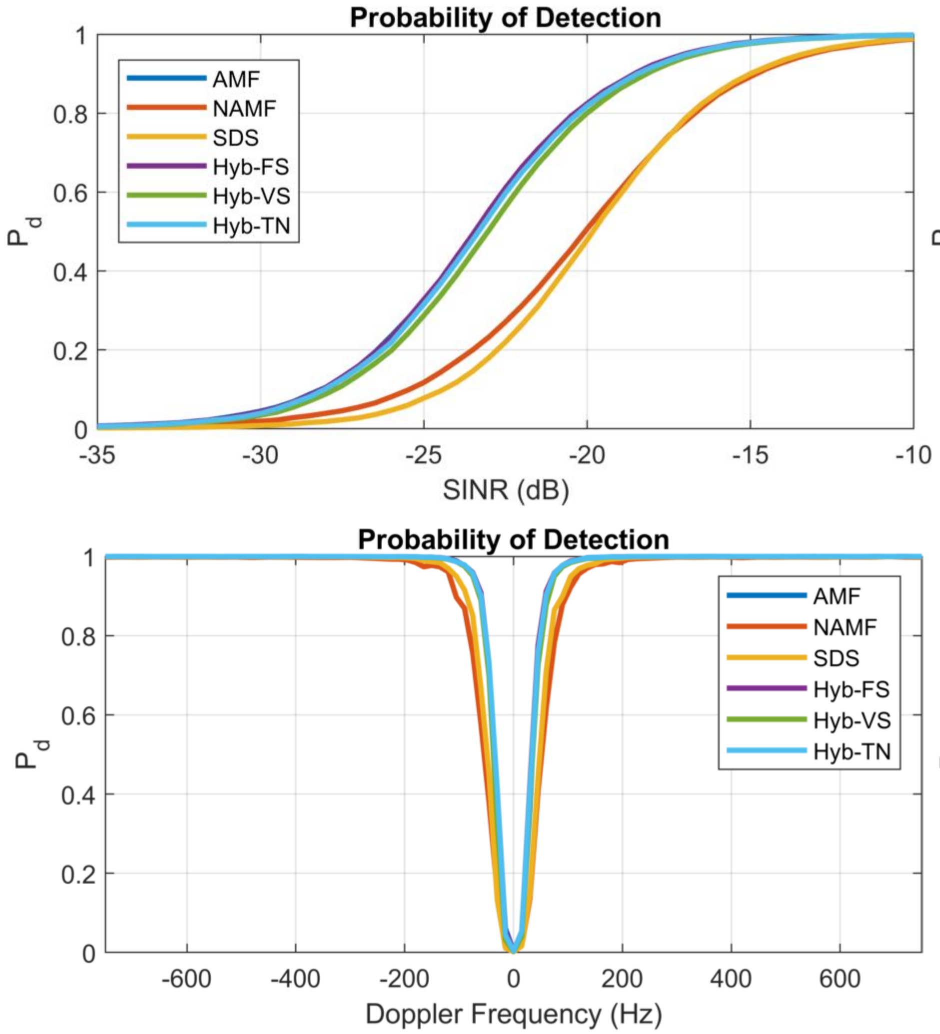
$$\mu = \frac{a^2}{b}$$

- Smaller a implies stronger heterogeneity
- CPI comprised 135 pulses
- Target injected at $0°$ azimuth and Doppler 50Hz (equivalent to approximately 20km/h)
- Simulations use a $P_{fa} = 10^{-3}$ and 10,000 Monte Carlo Runs

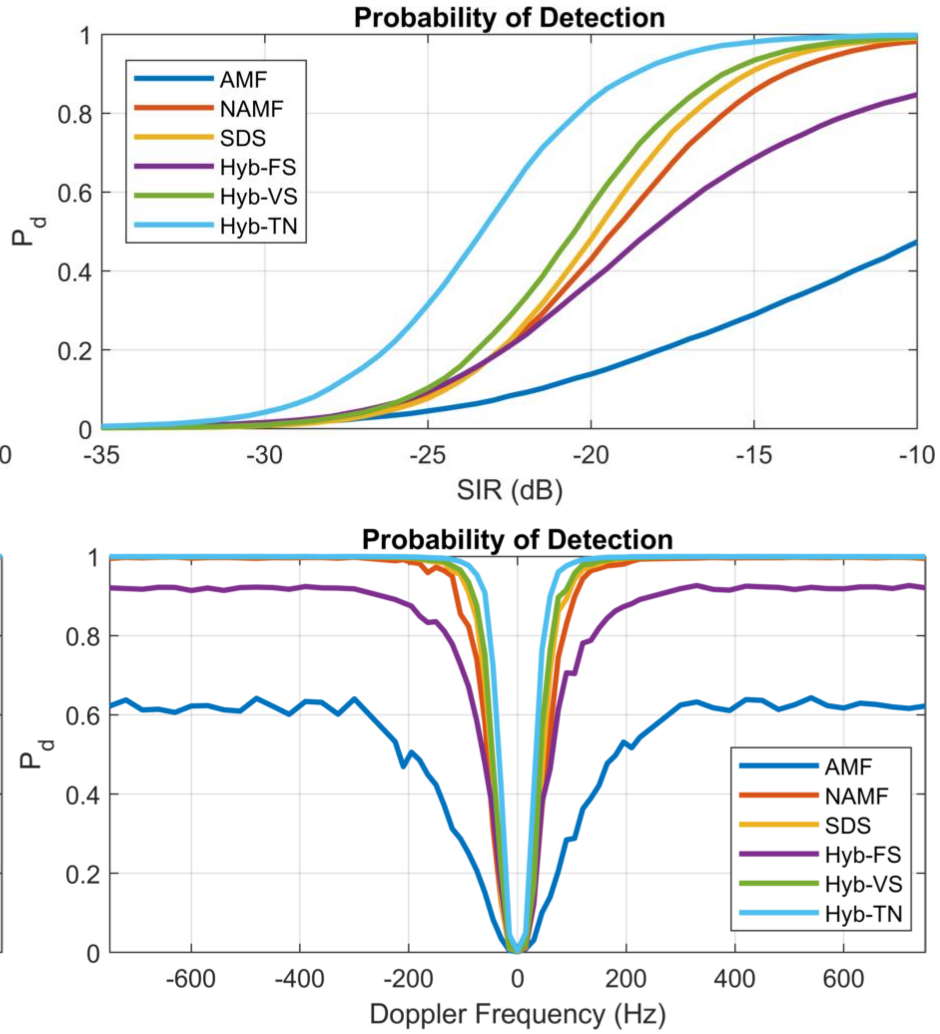
Parameter	Simulated	Ingara
Centre frequency (GHz)	1.33	1.33
Bandwidth (MHz)	140	140
Pulse repetition frequency (Hz)	1500	1500
Number of spatial channels	4	4
Inter-element spacing	$\lambda/2$	1.16λ
Aircraft speed (m/s)	100	88
Azimuth one-way 3 dB beamwidth	16°	13°
Clutter to noise ratio (dB)	20	10.1



Simulation Results



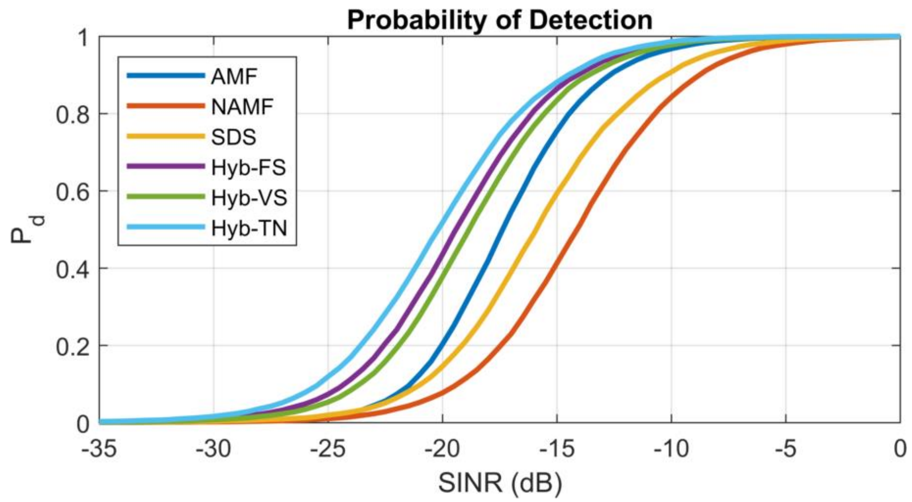
Homogeneous Environment, $a = 1000$

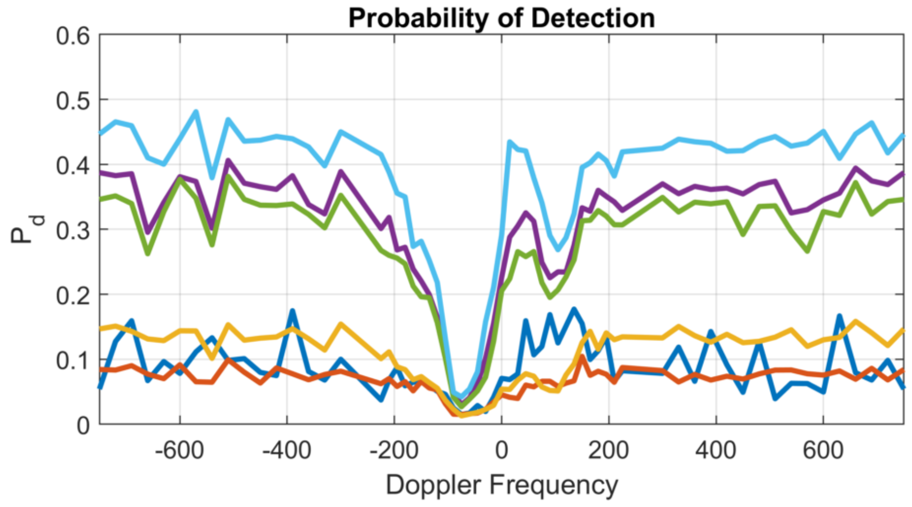


Heterogeneous Environment, $a = 1$

Experimental Results – Sea Clutter

- Ingara experimental data
- L-band radar with horizontal and vertical polarisations
- Horizontal polarisation with $30°$ grazing angle
- Douglas sea state between 3 and 4
- Shape parameter found to be 22.1 hence reasonably homogeneous data
- Target injected at $0°$ azimuth and Doppler 50Hz (approximately 20km/h)
- $P_{fa} = 10^{-3}$ and 10,000 Monte Carlo Runs





Reduced Dimension Detection

- Signal “lives” in a space of $L = NM$ dimensions but affords a more compact representation
- Brennan’s rule: clutter rank is $\leq M + \beta N - 1 \ll NM$
- Available number of degrees of freedom K

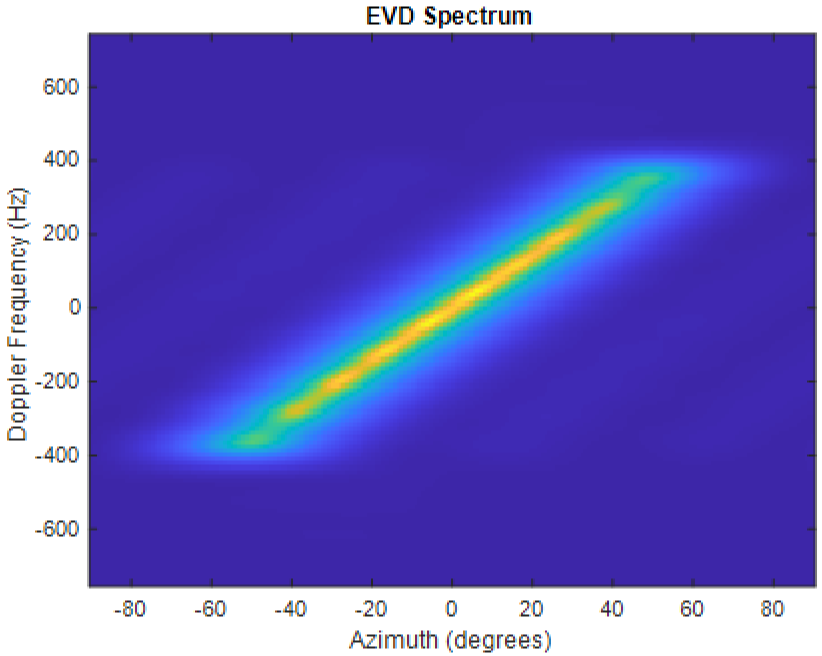
Number of degrees of freedom

$$K$$

Data dimensionality $\longleftarrow L$ $\quad$ $(K - L) \longrightarrow$ Detection reliability

- Reducing dimensionality to $D < L$
  - improves performance
  - reduces the training data requirements

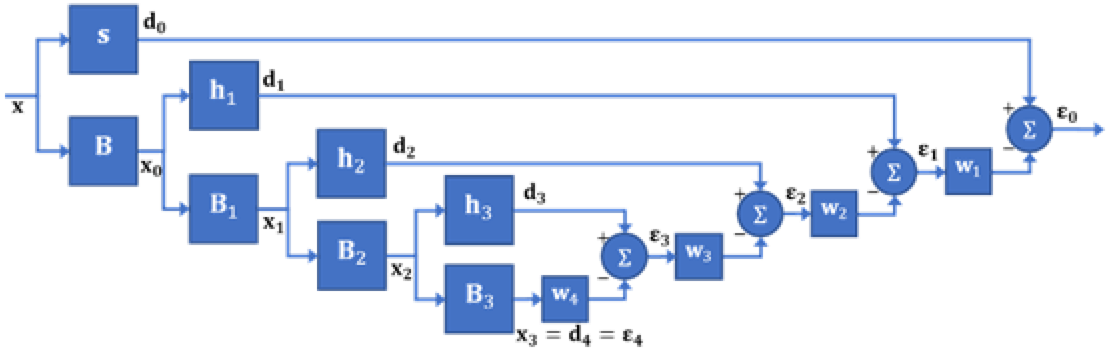
Reduced Rank Detectors



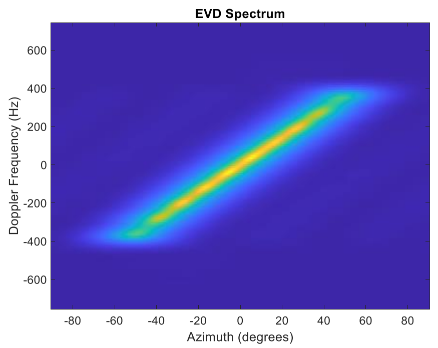
- What value for D? E.g. clutter rank?
- How to determine the new basis i.e. projection matrix $\mathbf{T}$?
- Principle Component Analysis
  - ➢ High clutter to noise ratio → clutter eigenvalues larger than noise eigenvalues
  - ➢ Separate the clutter subspace from noise subspace
  - ➢ Project onto the clutter subspace
- PCA aims to capture the entire clutter subspace
- Projection target-independent → Not necessarily compact
- Cross-spectral metric: computationally expensive

Multistage Wiener Filter

- Make target steering vector a dimension of the smaller subspace
- MWF decomposes data vector $\mathbf{x}$ through successive, nested Wiener filters
- Start with desired response $d_0 = \mathbf{s}^H \mathbf{x}$
- Find direction of maximum correlation with the desired response vector
- $$\mathbf{h}_\mathrm{i} = \mathbf{r}_{\mathbf{x}_{\mathrm{i}-1},\mathbf{d}_{\mathrm{i}-1}}$$
- Down-project onto the orthogonal subspace using $\mathbf{B} = \mathbf{I} - \mathbf{h}\mathbf{h}^\mathrm{H}$
- Iterate process finding direction of maximum correlation with previous stage
- Can be executed up to L stages or truncated to a smaller subspace

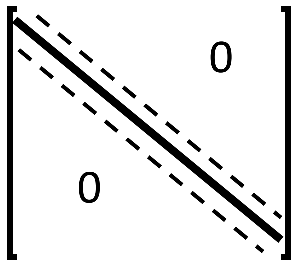


Why MWF?

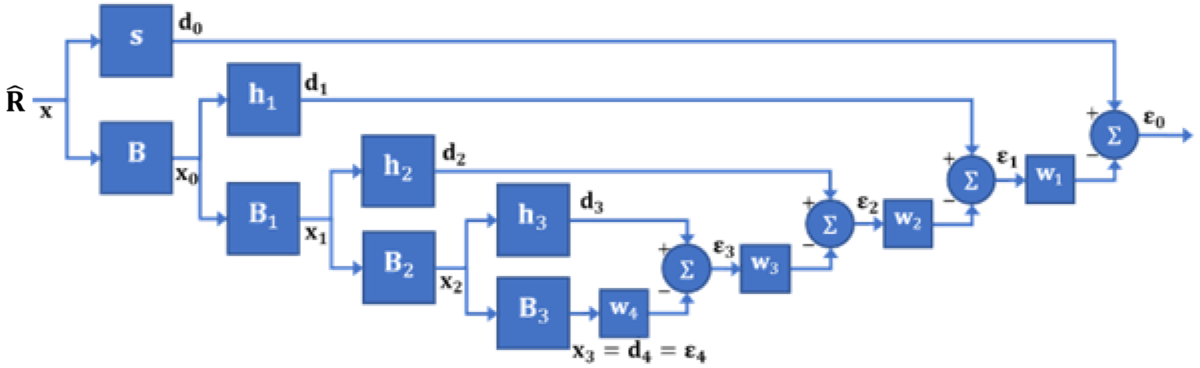


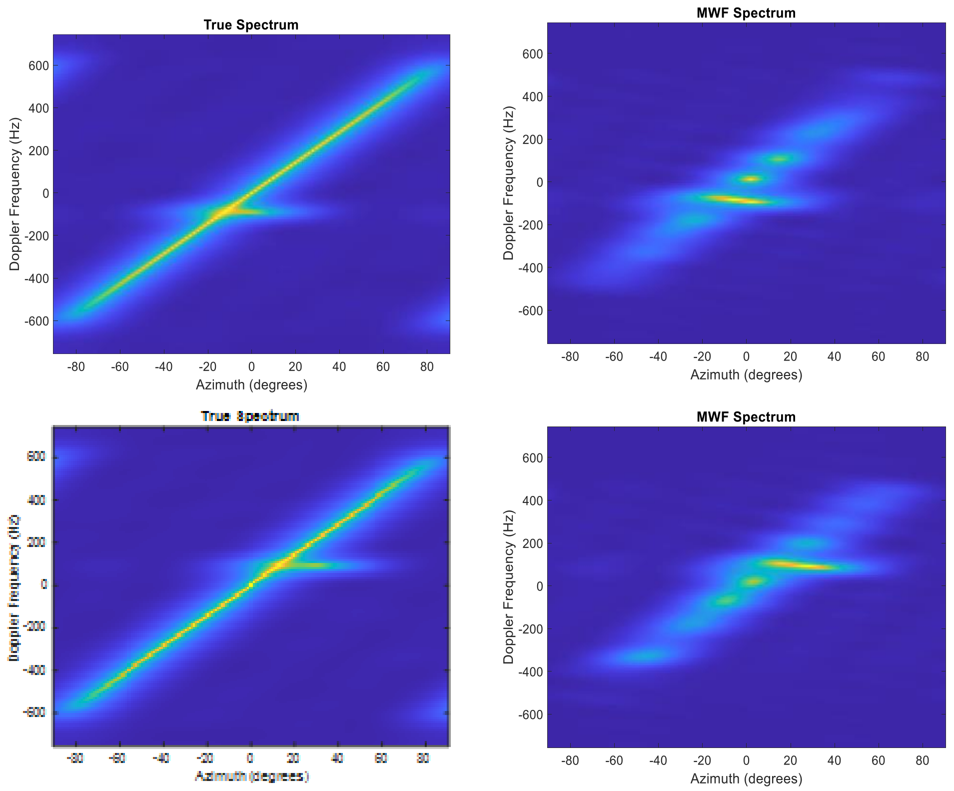
- MWF equivalently represented by transformation $\mathbf{T}$ that is
  - ➢ Constrained to have s as a basis vector
  - ➢ Tri-diagonalises the covariance matrix
- MWF is the solution of the problem

  Find the projection $\boldsymbol{T}$ that has $\boldsymbol{s}$ as a basis vector and tri-diagonalises the covariance matrix



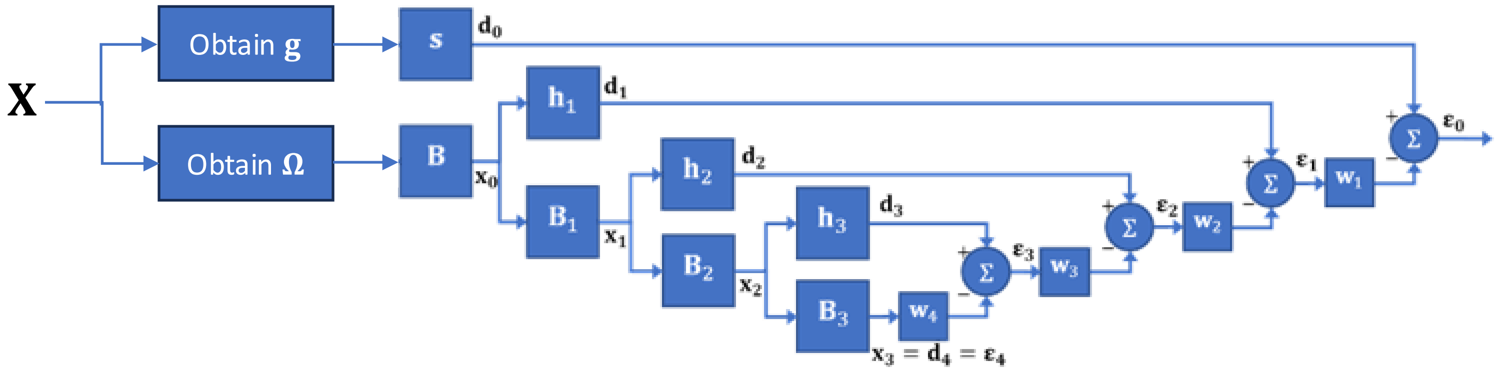
- MWF basis related to Krylov subspace





Reduced-Rank SDS and Hybrid Detectors

- Reduced rank SDS (SDS-R)
  - Combine the rank-reduction processing of the MWF with SDS
  - Treat $\mathbf{g}$ as the data vector $\mathbf{x}$ and $\mathbf{\Omega}$ as the covariance matrix estimate
- Assume $\mathbf{T}$ fixed to projection that diagonalises $\mathbf{R}$
  - derive the probabilities of false alarm and detection
  - show that the resulting detector is CFAR
- Actual performance shows loss due to $\mathbf{T}$ being a random variable



Reduced-Rank SDS and Hybrid Detectors

- Hybrid Reduced Rank FSH (FSH-R)
  - Input data vector $\mathbf{g}$
  - Input covariance matrix $\mathbf{\Sigma}_{\mathrm{FSH}} = \frac{1}{K_T+K_t-1}(\mathbf{W}\mathbf{W}^H - \mathbf{g}\mathbf{g}^H)$
- Hybrid Reduced Rank VSH (VSH-R)
  - Inversion of $\mathbf{\Omega}$ is unstable with low sample support
  - VSH-R overcomes this issue
  - First apply the MWF-SDS to give the down-projection matrix

$$\mathbf{T} = [\mathbf{s}\ \mathbf{h}_1\ \mathbf{h}_2\ \ldots \mathbf{h}_F]$$

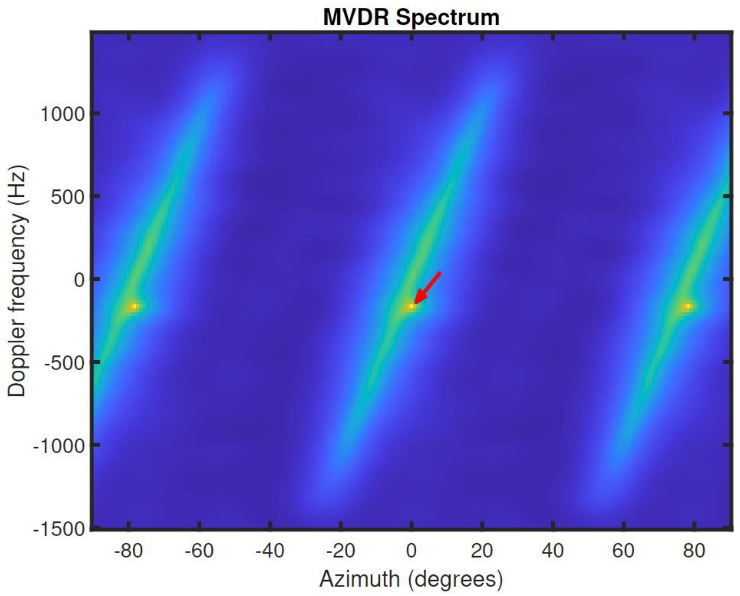
  - Down-project the CUT and training data

$$\mathbf{\Omega}_{\mathrm{r}} = \mathbf{T}^{\mathrm{H}}\mathbf{\Omega},\ \ \mathbf{z}_{\mathrm{r}} = \mathbf{T}^{\mathrm{H}}\mathbf{z},\ \ \mathbf{g}_{\mathrm{r}} = \mathbf{T}^{\mathrm{H}}\mathbf{g}$$

  - Apply the VSH to the rank reduced data

Data Sets and Simulation Parameters

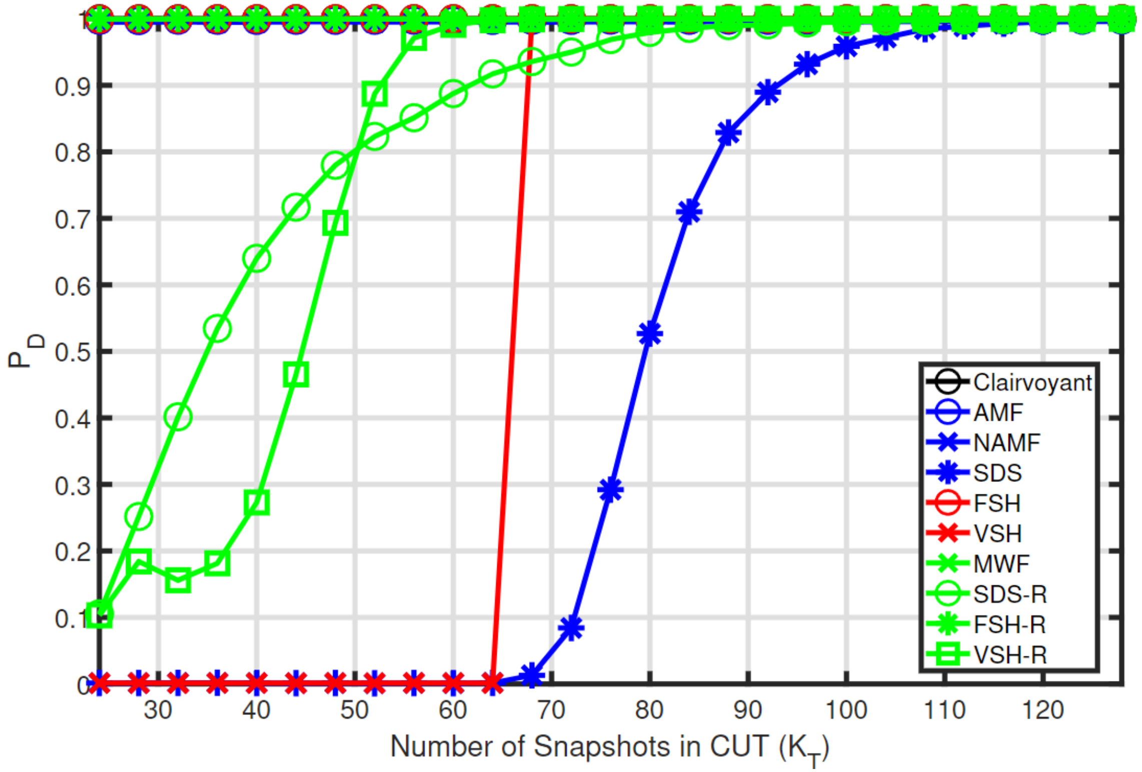
- Sea clutter simulation modelling the X-band Ingara sea-clutter dataset:
  - ➢Evolved Doppler spectrum with K-distributed clutter
  - ➢Upwind, 30º grazing angle and sea state 3
  - ➢Two scenarios: $a = 1000$ (homogenous) and $a = 0.2$ (heterogeneous)
- Side looking airborne radar with 4 spatial channels
- Swerling-1 target model
- MWF: number of stages (F) $\approx$ clutter rank
- SDS: $P = 16, Q = 4$, K_T varying but $K_t = 2PQ$
- Monte Carlo simulation performed $P_{\mathrm{FA}} = 10^{-3}$



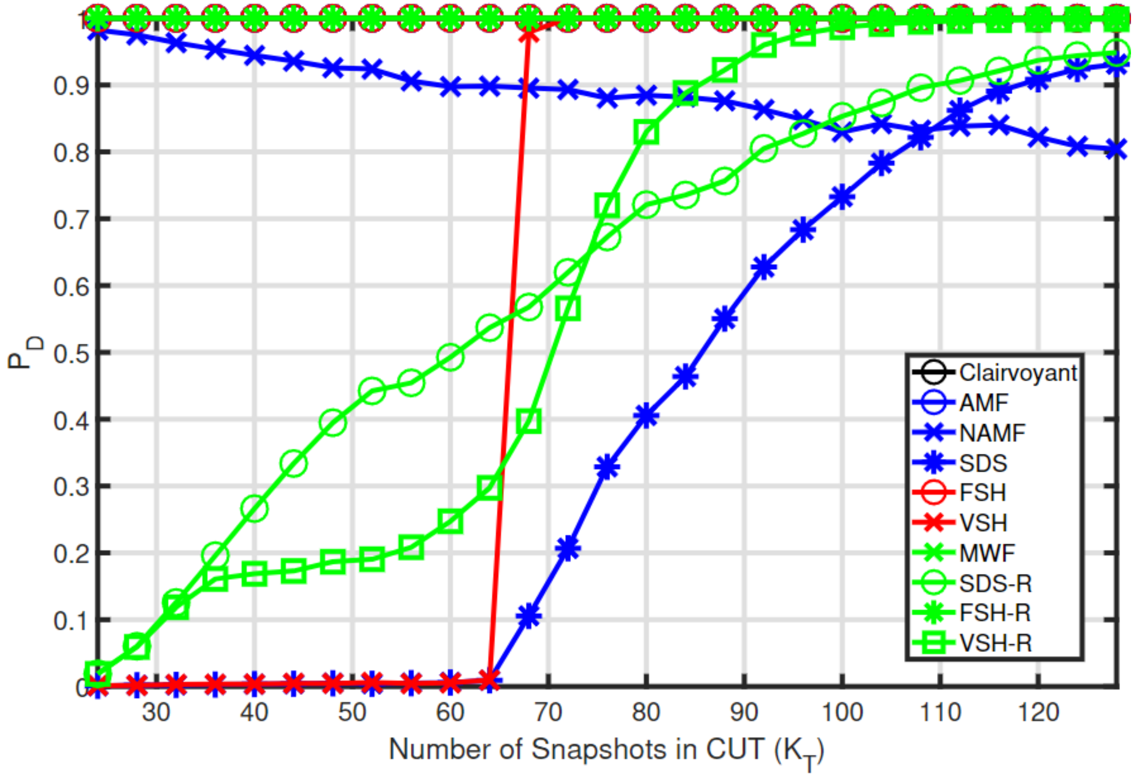
Parameter	Value
Carrier frequency, f_c	10 GHz
Bandwidth, B	200 MHz
Pulse repetition frequency, f_r	3000 Hz
Polarisation	Horizontal
Platform velocity, v_p	70 m/s
Azimuth two way 3 dB beamwidth	12.5°
Clutter to noise ratio	30 dB
Shape parameter, a	1000 and 0.2

Simulation Results

Homogeneous Clutter (a = 1000)



Independent Case

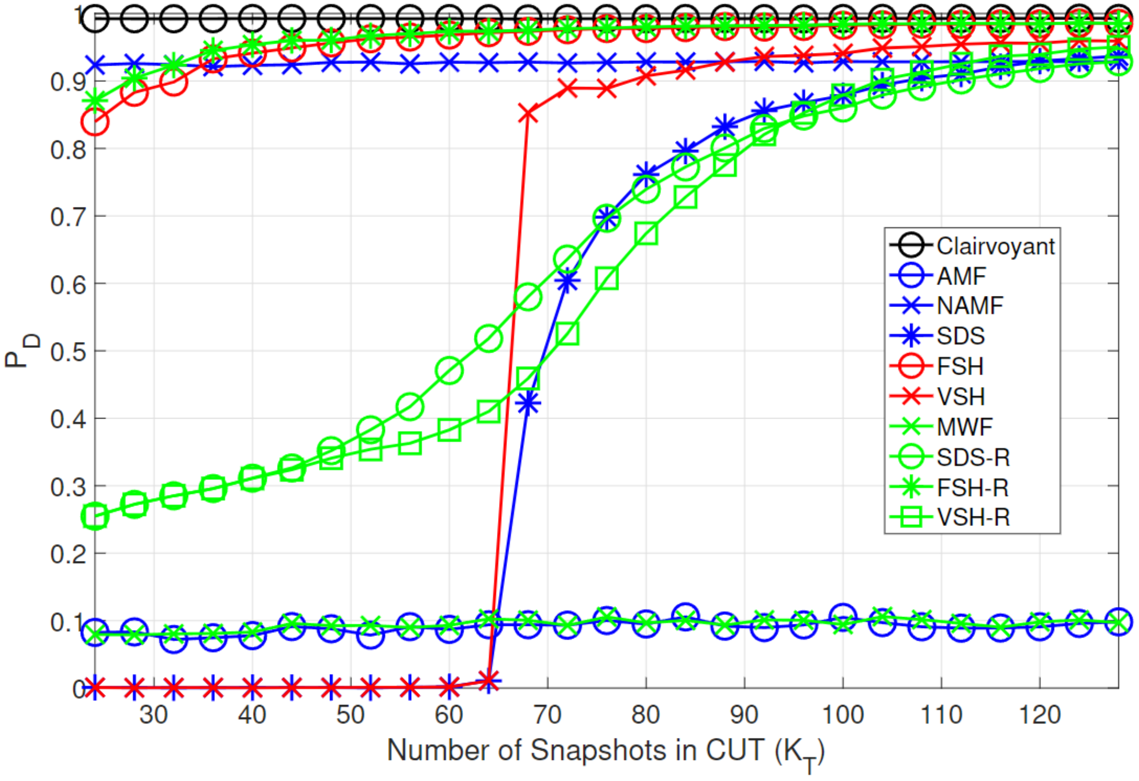


Partitioned Case

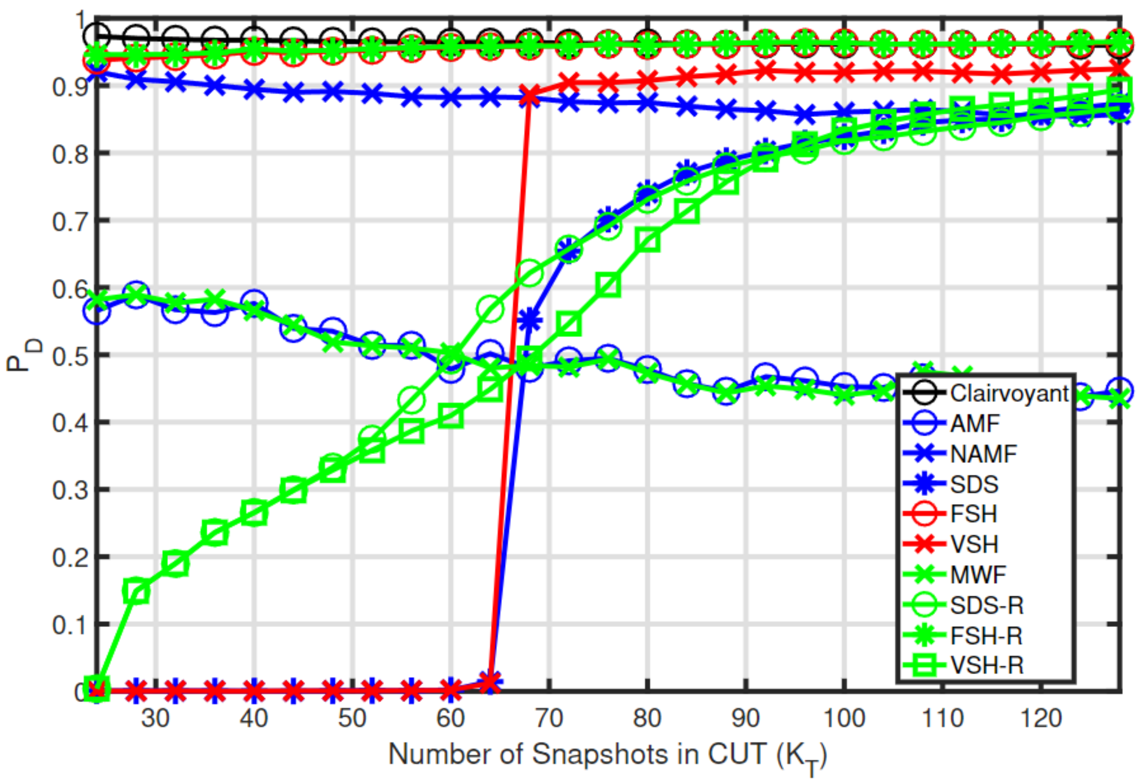
UNSW SYDNEY

Simulation Results

Heterogeneous Case (*a* = 0.2)



Independent Case



Partitioned Case

The Rank Estimation Problem

- Recall the question “What value for D? E.g. clutter rank?”
- Reduced rank techniques require the dimensionality of the subspace of interest
  - ➢ Underestimating the dimensionality $\rightarrow$ worse interference suppression
  - ➢ Overestimating the dimensionality $\rightarrow$ sample support requirement
- Require clutter rank
- Challenge: How to robustly estimate the clutter rank?
- Threshold based techniques
  - ➢ New Information
  - ➢ Ritz Value Estimation

Information Theoretic Criteria - MDL

- Avoid need user-defined threshold
- Trade the likelihood of the model against its cost
- Minimise the cost function

$$C(F) = -2\mathcal{L}\mathcal{L}(F) + p(F)$$

- Covariance matrix $\mathbf{R} = \mathbf{R}_\mathrm{c} + \sigma^2\mathbf{I}$, assuming $\mathbf{Z}$ is available

$$\mathcal{L}\mathcal{L}(F|\mathbf{Z}) = \ln\left(\frac{\prod_{i=F+1}^{L}\lambda_i^{\frac{1}{L-F}}}{\frac{1}{L-F}\sum_{i=F+1}^{L}\lambda_i}\right)^{(L-F)K}$$

- The penalty function accounts for the degrees of freedom (DOFs) of the model

$$p(F) = F(2L-F)\ln K$$

RVE-based MDL

- Aim to embed MDL into MWF
- RVEs are good approximations for the eigenvalues
- Can use RVE values in the MDL expression

$$\mathcal{LL}(F|\mathbf{Z}) = \ln\left(\frac{\prod_{i=F+1}^{L} \theta_i^{\frac{1}{L-F}}}{\frac{1}{L-F}\sum_{i=F+1}^{L} \theta_i}\right)^{(L-F)K}$$

- Eliminates need for threshold
- Requires full execution of MWF
- Need another way of embedding MDL to avoid full execution of MWF

Embedded MDL

- Desire to avoid eigen decomposition
- Embed MDL within MWF structure for truncation → evaluate cost function at each stage
- Need to avoid reliance on smallest eigenvalues (later stages)
- Arithmetic mean of smallest $(L-F)$-th eigenvalues obtained from trace of covariance matrix at the F-th stage

$$\sum_{i=F+1}^{L} \lambda_i = \mathrm{Tr}(\mathbf{R}_F)$$

- Produce and hence geometric mean smallest $(L-F)$-th eigenvalues expressed in terms of F largest eigenvalues

$$\prod_{i=F+1}^{L} \lambda_i = \frac{\det(\mathbf{R})}{\prod_{i=1}^{F} \lambda_i}$$

Embedded MDL

- Substituting into the log-likelihood

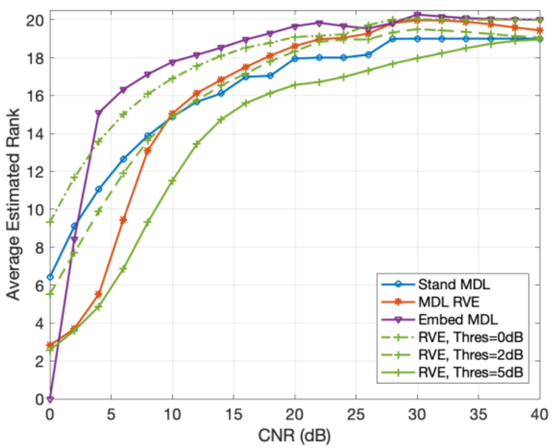
$$\mathcal{L}\mathcal{L}(F) = \ln\left(\frac{\det(\mathbf{R})}{\frac{1}{L-F}\operatorname{Tr}(\mathbf{R}_F)\prod_{i=1}^{F}\lambda_i}\right)^{(L-F)K}$$

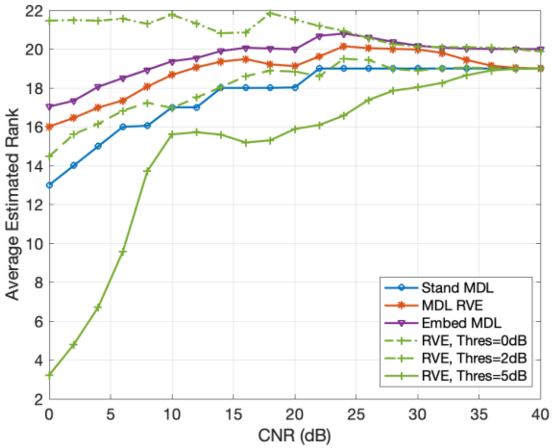
- Determinant of $\mathbf{R}$ constant and can be ignored
- The cost function is calculated at each stage
- Execution is continued until a turning point is observed
- Therefore, need $r+1$ stages where r is the clutter rank

Simulation Results

Parameter	Value
Carrier frequency, f_c	1.32 GHz
Pulse repetition frequency, f_r	1500 Hz
Platform velocity, v_p	85 m/s
Beam pattern	Cosine

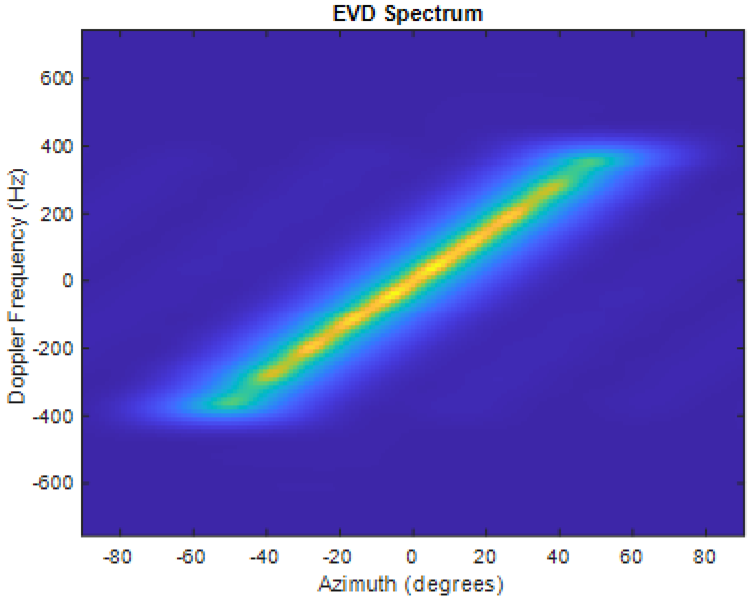
- N=4, M=16, 10,000 runs
- Brennan's rule rank 19
- CNRs from 0 to 40 dB
- Standard MDL: gives19 at high CNR
- RVE methods: impact of threshold 'tuning'
- Embedded MDL and MDL RVE slight overestimation at high CNR

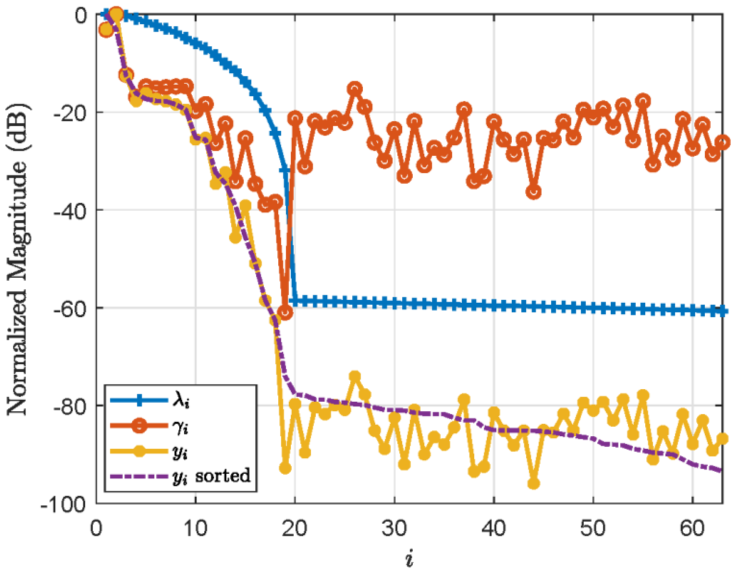




Target-Focused MWF Truncation

- Truncation is based on clutter rank
- Current implementations of MWF aim to capture entire clutter subspace
- Not all clutter is relevant to target direction
- Known that fewer stages than clutter rank needed for maximum P_d



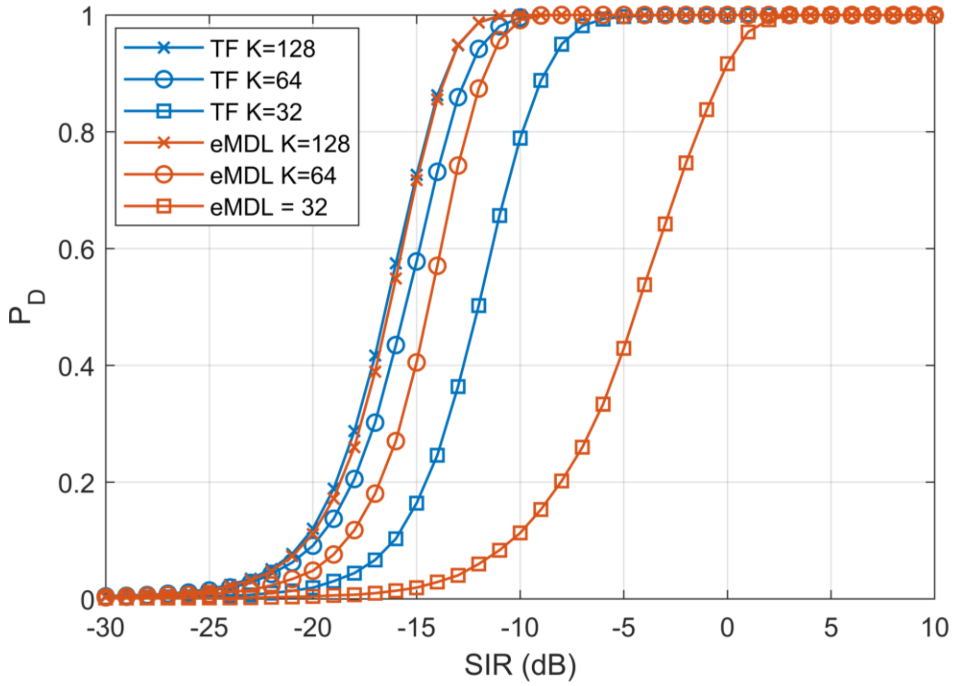


Problem Reformulation

- Current approaches estimate clutter rank
- Question that should be ask is”

  Which clutter dimensions actually matter to the target location

- MWF stage determination should seek to answer this question
- Derive an MDL-like solution
- Maximise information between recovered subspace and target signal



- Future work will analyse this approach and improve it

References

1. J. Rissanen, “Modeling by shortest data description,” Automatica, vol. 14, no. 5, pp. 465 – 471, 1978.
2. M. Wax and T. Kailath, “Detection of signals by information theoretic criteria,” IEEE Transactions on Acoustics, Speech, and Signal Processing, vol. 33, no. 2, pp. 387–392, 1985.
3. E. J. Kelly, “An adaptive detection algorithm,” IEEE Transactions on Aerospace and Electronic Systems, vol. AES-22, no. 1, pp. 115–127, 1986.
4. W. Chen and I. Reed, “A new CFAR detection test for radar,” Digital Signal Processing, vol. 1, no. 4, pp. 198–214, 1991.
5. J. S. Goldstein and I. S. Reed, “Reduced rank adaptive filtering,” IEEE Transactions on Signal Processing, vol. 45, no. 2, pp. 492–496, 1997.
6. F. Gini, M. V. Greco, A. Farina and P. Lombardo, "Optimum and mismatched detection against K-distributed plus Gaussian clutter," in IEEE Transactions on Aerospace and Electronic Systems, vol. 34, no. 3, pp. 860-876, July 1998, doi: 10.1109/7.705893.
7. J. S. Goldstein, I. S. Reed, and L. L. Scharf, “A multistage representation of the Wiener filter based on orthogonal projections,” IEEE Transactions on Information Theory, vol. 44, no. 7, pp. 2943–2959, 1998.
8. S. Kraut and L. L. Scharf, “The CFAR adaptive subspace detector is a scale-invariant GLRT,” IEEE Transactions on Signal Processing, vol. 47, no. 9, pp. 2538–2541, Sept 1999.
9. P. Lombardo, M. Greco, F. Gini, A. Farina and J. B. Billingsley, "Impact of clutter spectra on radar performance prediction," in IEEE Transactions on Aerospace and Electronic Systems, vol. 37, no. 3, pp. 1022-1038, July 2001, doi: 10.1109/7.953265.1.2160861.
10. M. Rangaswamy, J. Michels, and B. Himed, “Statistical analysis of the non-homogeneity detector for STAP applications,” Digital Signal Processing: A Review Journal, vol. 14, no. 3, pp. 253–257, May 2004.
11. S. Mangiat, K. Y. Li, S. Unnikrishna Pillai and B. Himed, "Effect of terrain modeling and internal clutter motion on space based radar performance," 2006 IEEE Conference on Radar, Verona, NY, USA, 2006, pp. 8 pp.-, doi: 10.1109/RADAR.2006.1631817.
12. P. Wang, H. Li and B. Himed, "Moving Target Detection Using Distributed MIMO Radar in Clutter With Nonhomogeneous Power," in IEEE Transactions on Signal Processing, vol. 59, no. 10, pp. 4809-4820, Oct. 2011, doi: 10.1109/TSP.2011.2160861.

References

13. V. Gracheva and J. Ender, "Multichannel Analysis and Suppression of Sea Clutter for Airborne Microwave Radar Systems," in IEEE Transactions on Geoscience and Remote Sensing, vol. 54, no. 4, pp. 2385-2399, April 2016, doi: 10.1109/TGRS.2015.2500918

14. M. Zhang, A. Zhang, and J. Li, “Fast and accurate rank selection methods for multistage wiener filter,” IEEE Transactions on Signal Processing, vol. 64, no. 4, pp. 973–984, 2016.

15. A. De Maio, "Invariance Theory for Adaptive Radar Detection in Heterogeneous Environment," in IEEE Signal Processing Letters, vol. 26, no. 7, pp. 996-1000, July 2019, doi: 10.1109/LSP.2019.2916749.

UNSW SYDNEY

Thank you

Questions?

elias@ieee.org